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MUTUAL ADMITTANCE BETWEEN SLOTS ON A CYLINDER,

(10)
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Y(12)

In the design of conformal slot array on the surface of a conducting cylinder, the calculation of the mutual admittance Y_{12} is a crucial step, which has been studied extensively in recent years. ~~In~~ this paper, ~~we~~ summarizes, in a handbook format, all of the final formulas of Y_{12}, and presents some typical numerical data.

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by

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1. INTRODUCTION

In the design of a conformal slot array on the surface of a conducting cylinder, the calculation of the mutual admittance Y_{12} is a crucial step, which has been studied extensively in recent years. In this paper, we summarize, in a handbook format, all of the final formulas of Y_{12}, and present some typical numerical data.

2. STATEMENT OF PROBLEM

Referring to Figure 1, two identical slots, circumferential or axial, are located on the surface of an infinitely long cylinder. The geometrical parameters are

$$R = \text{radius of the cylinder} \tag{2.1}$$

$$(a,b) = \text{dimensions of the slot along } (\phi,z) \text{ directions (a is the arc length along the cylinder)} \tag{2.2}$$

$$(z_0, R\phi_0) = \text{center-to-center distances between slots} \tag{2.3}$$

$$s_0 = \sqrt{z_0^2 + (R\phi_0)^2} \tag{2.4}$$

$$\theta_0 = \tan^{-1}(z_0/R\phi_0) \tag{2.5}$$

The problem is to determine the mutual admittance between these two slots when kR is large.

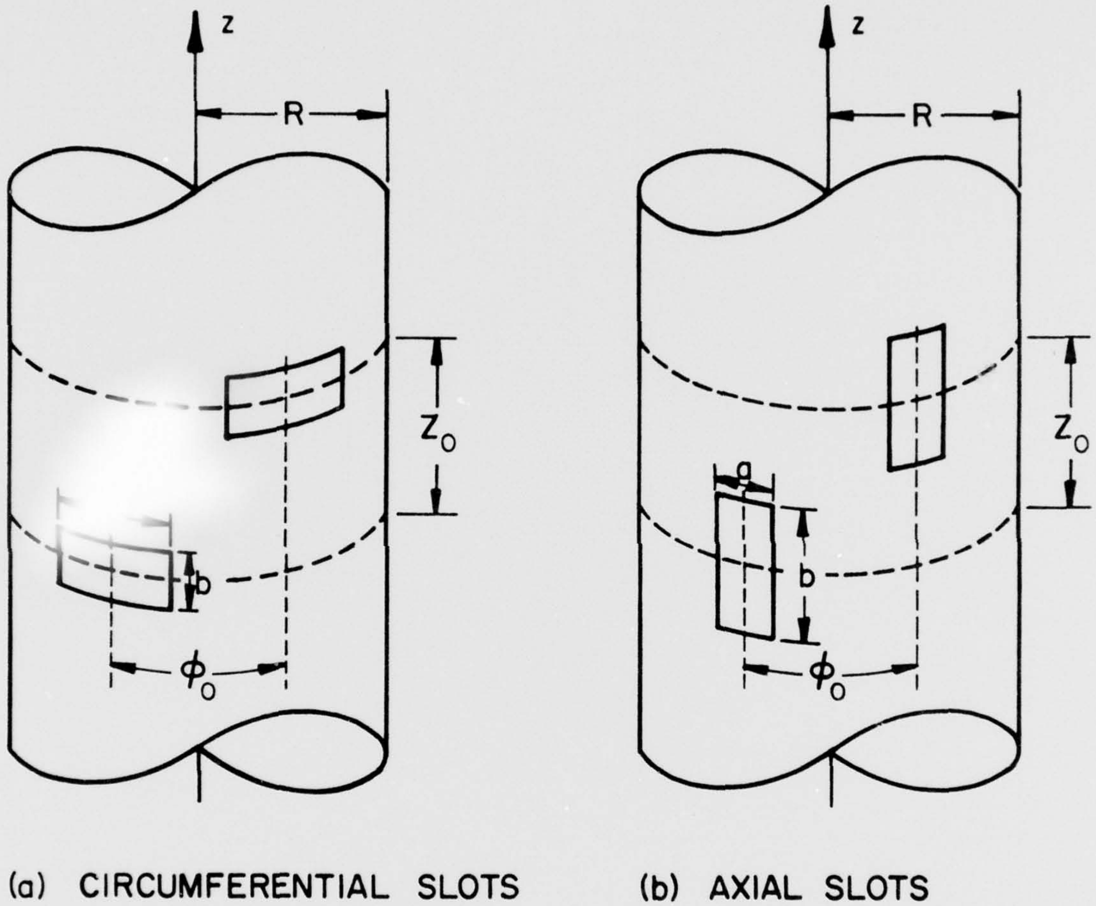

Figure 1. Two identical slots on the surface of a cylinder.

First let us define mutual admittance. Throughout this work we always assume that

(i) the slots are thin, and (2.6a)

(ii) their length is roughly a half-wavelength. (2.6b)

Then the aperture field in each slot can be adequately approximated by a simple cosine distribution, which is the so-called "one-mode" approximation. For example. if slot 1 is a circumferential (lower slot in Figure 1a), its aperture field under the "one-mode" approximation is given by (exp + $j\omega t$ time convention)

$$\vec{E} = V_1 \vec{e}_1 \ , \ \vec{H} = I_1 \vec{h}_1 \tag{2.7a}$$

where

$$\vec{e}_1 = \hat{z}\sqrt{\frac{2}{ab}} \cos \frac{\pi}{a} y \ , \ \vec{h}_1 = \hat{x} \times \vec{e}_1 \tag{2.7b}$$

$$y = R\phi \ . \tag{2.7c}$$

(V_1, I_1) are respectively the modal (voltage, current) of slot 1. The mutual admittance Y_{12} is defined by

$$Y_{12} = Y_{21} = \frac{I_{21}}{V_1} \tag{2.8}$$

where I_{21} is the induced current in slot 2 when slot 1 is excited by a voltage V_1 and slot 2 is short-circuited. An alternative expression for Y_{12} is

$$Y_{12} = \frac{1}{V_1 V_2} \iint_{A_2} \vec{E}_2 \times \vec{H}_1 \cdot d\vec{s}_2 \tag{2.9}$$

where

A_2 = aperture of slot 2

$\vec{H}_1$ = magnetic field when slot 1 is excited with voltage V_1, and slot 2 is covered by a perfect conductor

$\vec{E}_2$ = electric field when slot 2 is excited with voltage V_2, and slot 1 is covered by a perfect conductor.

Because $\vec{H}_1 = I_{21}\vec{h}_2$ and $\vec{E}_2 = V_2\vec{e}_2$, it is a simple matter to verify that (2.8) and (2.9) are equivalent [1].

There is an alternative definition of mutual admittance. Instead of (2.7), a modal voltage $\overline{V}_1$ (with a bar) may be defined through the expression for the aperture field of slot 1 as follows:

$$\vec{E} = \hat{z}\,\frac{1}{b}\,\overline{V}_1 \cos\frac{\pi}{a}\,y \qquad (2.10a)$$

or equivalently

$$\overline{V}_1 = \int_{-b/2}^{b/2} (\hat{z}\cdot\vec{E})_{y=0}\,dz \quad . \qquad (2.10b)$$

Then a different mutual admittance $\overline{Y}_{12}$ is defined by (2.9) after replacing (V_1, V_2) by $(\overline{V}_1, \overline{V}_2)$. It can be easily shown that

$$\overline{Y}_{12} = \frac{a}{2b}\,Y_{12} \quad . \qquad (2.11)$$

Two remarks are in order: (i) In the limiting case that $b \to 0$, Y_{12} goes to zero as b , whereas $\overline{Y}_{12}$ approaches a constant independent of b. (ii) For the special case $a = \lambda/2$ and $R \to \infty$, it is $\overline{Y}_{12}$ that is identical to the mutual impedance Z_{12} between two corresponding dipoles calculated by the classical Carter's method [2], [3], [4]. (iii) When the slots are excited by waveguides (transmission lines), one often uses Y_{12} $(\overline{Y}_{12})$. From here on, we will concentrate on Y_{12} instead of $\overline{Y}_{12}$.

The mutual admittance defined in (2.8) and (2.9) includes the self admittance Y_{11} as a special case which occurs when two slots coincide. (All the formulas of Y_{12} given in this paper, except for the one in Section 4, can be used for calculating Y_{11} by setting $\phi_0 \to 0$ and $z_0 \to 0$.)

3. EXACT HUGHES (GSP) MODAL SOLUTION

Once the one-mode approximation in (2.7) is accepted, Y_{12} can be determined exactly in terms of cylindrical modal functions, as has been done by Stewart, Golden, and Pridmore-Brown [5], [6]. The final result reads:

Circumferential slots

$$Y_{12} = \int_{-\infty}^{\infty} dk_z \sum_{m=-\infty}^{\infty} \psi(m,k_z)G(m,k_z)e^{-j(m\phi_0+k_z z_0)} \tag{3.1}$$

where

$$\psi(m,k_z) = \frac{ab}{8\pi^2 R} \frac{\sin^2(k_z b/2)}{(k_z b/2)^2} \cdot \left\{\frac{\sin(m\phi_a + \pi/2)}{(m\phi_a + \pi/2)} + \frac{\sin(m\phi_a - \pi/2)}{(m\phi_a - \pi/2)}\right\}^2 \tag{3.2}$$

$$\phi_a = (a/2R)$$

$$G(m,k_z) = Y_0\left[\frac{jk}{k_t} \frac{H_m^{(2)'}(k_t R)}{H_m^{(2)}(k_t R)} + \left(\frac{mk_z}{k_t^2 R}\right)^2 \frac{k_t}{jk} \frac{H_m^{(2)}(k_t R)}{H_m^{(2)'}(k_t R)}\right] . \tag{3.3}$$

$$k_t = \begin{cases} \sqrt{k^2 - k_z^2} & , \text{ if } k \geq k_z \\ -j\sqrt{k_z^2 - k^2} & , \text{ if } k \leq k_z \end{cases} \tag{3.4}$$

Axial slots

$$Y_{12} = \int_{-\infty}^{\infty} dk_z \sum_{m=-\infty}^{\infty} \phi(m,k_z)\, F(m,k_z)e^{-j(m\phi_0 + k_z z_0)} \tag{3.5}$$

where

$$\phi(m,k_z) = \frac{ab}{8R}\left(\frac{\sin(m\phi_a)}{(m\phi_a)} \cdot \frac{\cos(k_z b/2)}{(k_z b/2)^2 - (\pi/2)^2}\right)^2 \tag{3.6}$$

$$F(m,k_z) = Y_0 \frac{k_t}{jk} \frac{H_m^{(2)}(k_t R)}{H_m^{(2)'}(k_t R)} \tag{3.7}$$

This solution is suitable for numerical calculation if (i) $z_0 < b$ for circumferential slots, and $z_0 < a$ for axial slots, (ii) kR is less than 20, and (iii) the medium is slightly lossy so that k has a small (negative) imaginary part. Based on this solution, extensive numerical results have been reported by Hughes Aircraft Company at Culver City [7], [8], [9].

4. EXACT UI MODAL SOLUTION

Under the one-mode approximation, another exact modal solution is given in [10]. This solution is derived from the Hughes (SGP) solution in Section 3 by a deformation of integration contour and an application of the Duncan transform [11]. THe final result reads

Circumferential slots

$$Y_{12} = G + jB \tag{4.1a}$$

$$G = \int_0^k \sum_{m=0}^{\infty} \frac{\cos m\phi_0}{\varepsilon_m} \cos k_z z_0 \; \psi(m,k_z) R(m,k_z) \; dk_z \tag{4.1b}$$

$$B = \sum_{m=0}^{\infty} \frac{\cos m\phi_0}{\varepsilon_m} \left\{ -\int_0^k R(m,k_z)\psi(m,k_z) \sin k_z z_0 \; dk_z + \int_0^{\infty} R(m,j\eta)\psi(m,j\eta) e^{-\eta z_0} \, d\eta \right\} . \tag{4.1c}$$

where

$$R(m,k_z) = \frac{2}{\pi k_t R} \cdot \frac{k}{k_t} \cdot \left[\frac{1}{M_m^2(k_t R)} + \left(\frac{mk_z}{k_t k R} \right)^2 \frac{1}{N_m^2(k_t R)} \right] \tag{4.2}$$

$$M_m^2(\chi) = J_m^2(\chi) + Y_m^2(\chi) \tag{4.3}$$

$$N_m^2(\chi) = J_m'^2(\chi) + Y_m'^2(\chi) \tag{4.4}$$

$$\varepsilon_m = \begin{cases} 2 \; , & m = 0 \\ 1 \; , & m \neq 0 \end{cases} . \tag{4.5}$$

$\psi(m,k_z)$ is defined in (3.2) and k_t in (3.4) (4.6)

Axial slots

$$Y_{12} = \frac{8Y_0}{\pi kR} \sum_{m=0}^{\infty} \frac{\cos m\phi_0}{\varepsilon_m} \left(\int_0^k \Phi(m,k_z) e^{-jk_z z_0} \frac{dk_z}{N_m^2(k_t R)} \right.$$

$$\left. + j \int_0^{\infty} \Phi(m,j\eta) e^{-\eta z_0} \frac{d\eta}{N_m^2\left(R\sqrt{\eta^2 + k^2}\right)} \right) \tag{4.7}$$

where $\Phi(m,k_z)$ is defined in (3.5)

This solution is valid only if $z_0 > b$ for circumferential slots and $z_0 > a$ for axial slots. It is suitable for numerical calculation if kR is less than 20.

5. ASYMPTOTIC SOLUTION

The two modal solutions given in Sections 3 and 4 are based on fields in the Fourier transform domain. An alternative calculation of Y_{12} involves the field in the spatial domain, namely,

Circumferential slots

$$Y_{12} = \frac{-2}{ab} \int_{A_1} dy_1 dz_1 \int_{A_2} dy_2 dz_2 \, [\cos \tfrac{\pi}{a} y_1][\cos \tfrac{\pi}{a}(y_2 - R\phi_0)] g_\phi(s,\theta) \tag{5.1}$$

Axial slots

$$Y_{12} = \frac{-2}{ab} \int_{A_1} dy_1 dz_1 \int_{A_2} dy_2 dz_2 \, [\cos \tfrac{\pi}{b} z_1][\cos \tfrac{\pi}{b}(z_2 - z_0)] g_z(s,\theta) \tag{5.2}$$

where (y_n, z_n) = a typical point in the aperture of slot n (n = 1 or 2). (5.3)

A_n = *aperture of slot* n (5.4)

$$s = \sqrt{(y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{5.5}$$

$$\theta = \tan^{-1}[(z_2 - z_1)/(y_2 - y_1)] \tag{5.6}$$

Several versions of the Green's functions g_ϕ and g_z have been approximately determined under the condition that $kR >> 1$. They are listed as follows:

<u>OSU Asymptotic solution</u> [12] [13]

$$g_\phi \sim G(s)\ [v(\xi)\ \sin^2\theta + (\frac{j}{ks})u(\xi)\ \cos^2\theta] \tag{5.7}$$

$$g_z \sim G(s)\ [v(\xi)\ \cos^2\theta + (\frac{j}{ks})u(\xi)\ \sin^2\theta] \tag{5.8}$$

<u>PINY Asymptotic solution</u> [9] [14]

$$g_\phi \sim G(s)\Big\{v(\xi)[\sin^2\theta + \frac{j}{ks}\ (1 - 3\ \sin^2\theta)] + \frac{j}{ks}\ \sec^2\theta[u(\xi) - v_1(\xi)\ \sin^2\theta]\Big\} \tag{5.9}$$

$$g_z \sim G(s)\ v(\xi)\ [\cos^2\theta + \frac{j}{ks}\ (2 - 3\ \cos^2\theta)] \tag{5.10}$$

<u>UI Asymptotic solution</u> [15]

$$g_\phi \sim G(s)\Big\{v(\xi)[\sin^2\theta + \frac{j}{ks}\ \cos2\theta] + (\frac{j}{ks})\ u(\xi)[\cos^2\theta(1 - \frac{2j}{ks}) + (\frac{j}{ks})\ \sin^2\theta] + j(\sqrt{2}\ kR/\cos^2\theta)^{-2/3}[v'(\xi)\ \sin^2\theta + (\tan^4\theta + \frac{j}{ks})u'(\xi)\ \cos^2\theta]\Big\} \tag{5.11}$$

$$g_z = G(s)\Big\{v(\xi)[\cos^2\theta - \frac{j}{ks}\ \cos2\theta] + (\frac{j}{ks})u(\xi)[\sin^2\theta(1 - \frac{2j}{ks}) + (\frac{j}{ks})\ \cos^2\theta] + j(\sqrt{2}\ kR/\cos^2\theta)^{-2/3}[v'(\xi)\cos^2\theta + (1 + \frac{j}{ks})u'(\xi)\sin^2\theta]\Big\} \tag{5.12}$$

where

$$G(s) = \frac{k^2 Y_0}{2\pi j}\ \frac{e^{-jks}}{ks}\ ,\ Y_0 = (120\pi)^{-1} \tag{5.13}$$

$$\xi = (k\ \cos^4\theta/2R^2)^{1/3}s \tag{5.14}$$

The Fock functions, u, v, etc., can be calculated from the following two representations:

<u>For $0 \le \xi \le 0.7$</u>

$$v(\xi) \sim 1 - \frac{\sqrt{\pi}}{4} e^{j\pi/4}\xi^{3/2} + \frac{7j}{60}\xi^3 + \frac{7\sqrt{\pi}}{512} e^{-j\pi/4}\xi^{9/2} - 4.141 \times 10^{-3}\xi^6 \quad (5.15)$$

$$u(\xi) \sim 1 - \frac{\sqrt{\pi}}{2} e^{j\pi/4}\xi^{3/2} + \frac{5j}{12}\xi^3 + \frac{5\sqrt{\pi}}{64} e^{-j\pi/4}\xi^{9/2} - 3.701 \times 10^{-2}\xi^6 \quad (5.16)$$

$$v_1(\xi) \sim 1 + \frac{\sqrt{\pi}}{2} e^{j\pi/4}\xi^{3/2} - \frac{7j}{12}\xi^3 - \frac{7\sqrt{\pi}}{64} e^{-j\pi/4}\xi^{9/2} + 4.555 \times 10^{-2}\xi^6 \quad (5.17)$$

$$v'(\xi) \sim \frac{3\sqrt{\pi}}{8} e^{-j3\pi/4}\xi^{1/2} + \frac{7j}{20}\xi^2 + \frac{63\sqrt{\pi}}{1024} e^{-j\pi/4}\xi^{7/2} - 2.485 \times 10^{-2}\xi^5 \quad (5.18)$$

$$u'(\xi) \sim \frac{3}{4}\sqrt{\pi}\, e^{-j3\pi/4}\xi^{1/2} + \frac{5j}{4}\xi^2 + \frac{45\sqrt{\pi}}{128} e^{-j\pi/4}\xi^{7/2} - 2.221 \times 10^{-1}\xi^5 \quad (5.19)$$

<u>For $0.7 \le \xi \le \infty$</u>

$$v(\xi) \approx e^{-j\pi/4}\sqrt{\pi}\,\xi^{1/2} \sum_{n=1}^{10} (t'_n)^{-1} e^{-j\xi t'_n} \quad (5.20)$$

$$u(\xi) \approx e^{j\pi/4} 2\sqrt{\pi}\,\xi^{3/2} \sum_{n=1}^{10} e^{-j\xi t_n} \quad (5.21)$$

$$v_1(\xi) \approx e^{j\pi/4} 2\sqrt{\pi}\,\xi^{3/2} \sum_{n=1}^{10} e^{-j\xi t'_n} \quad (5.22)$$

$$v'(\xi) \approx \frac{1}{2} e^{-j\pi/4}\sqrt{\pi}\,\xi^{-1/2} \sum_{n=1}^{10} (1 - j2\xi t'_n)(t'_n)^{-1} e^{-j\xi t'_n} \quad (5.23)$$

$$u'(\xi) \approx e^{j\pi/4} 3\sqrt{\pi}\,\xi^{1/2} \sum_{n=1}^{10} \left(1 - j\frac{2}{3}\xi t_n\right) e^{-j\xi t_n} . \quad (5.24)$$

where $t_n = |t_n| \exp(-j\pi/3)$, $t'_n = |t'_n| \exp(-j\pi/3)$, and

n	$\lvert t_n \rvert$	$\lvert t'_n \rvert$
1	2.33811	1.01879
2	4.08795	3.24820
3	5.52056	4.82010
4	6.78671	6.16331
5	7.99413	7.37218

n	$\lvert t_n \rvert$	$\lvert t'_n \rvert$
6	9.02265	8.48849
7	10.04017	9.53545
8	11.00852	10.52766
9	11.93602	11.47506
10	12.82878	12.38479

It has been verified through several hundred numerical examples that the UI asymptotic solution given above is in excellent agreement (within a quarter db in magnitude and a few degrees in phase) with the exact model solution for all slot separations (ϕ_0, z_0) provided that $kR \geq 5$.

In using the asymptotic solutions for calculating the self admittance Y_{11}, care must be exercised in avoiding the singularity in the Green's function which occurs at $s = 0$. A most convenient way to avoid this apparent difficulty is to (i) use a large number of points for the two surface integrals in (5.1) and (5.2), and (ii) shift slightly the integration nets for this two surface integrals.

6. EXACT PLANAR SOLUTION

In the limit $kR \to \infty$ the Green's function of the UI solution in (5.11) and (5.12) is reduced to

$$g_\phi = G(s)[\sin^2\theta + \frac{j}{ks}(2 - 3\sin^2\theta)(1 - \frac{j}{ks})] \tag{6.1}$$

$$g_z = G(s)[\cos^2\theta + \frac{j}{ks}(2 - 3\cos^2\theta)(1 - \frac{j}{ks})] \quad . \tag{6.2}$$

When (6.1) and (6.2) are used in (5.1) and (5.2), we obtain the exact solution (under the "one-mode" approximation of course) for two slots on an infinitely large, conducting plane.

7. APPROXIMATE SOLUTION

Based on the UI asymptotic solution, a simple approximate solution, is reported in [10], i.e.,

Circumferential slots

$$Y_{12} \simeq -\frac{8ab}{\pi^2}[S(b\sin\theta)\, C(a\sin\theta)]^2\, \bar{g}_\phi \quad . \tag{7.1}$$

Axial slots

$$Y_{12} \simeq -\frac{8ab}{\pi^2}\,[S(a\cos\theta)\,C(b\sin\theta)]^2\,\bar{g}_z \tag{7.2}$$

where

$$S(x) = \frac{\sin(kx/2)}{(kx/2)}\ , \quad C(x) = \frac{\cos(kx/2)}{1-(kx/\pi)^2}\ . \tag{7.3}$$

The (simplified) Green's functions $\bar{g}_\phi$ and $\bar{g}_z$ are given by

$$\bar{g}_\phi = G(s)\Big[v(\xi)\Big(\sin^2\theta + \frac{j}{ks}\cos 2\theta\Big) + \frac{j}{ks}\,u(\xi)\cos^2\theta$$
$$+ ju'(\xi)(\sqrt{2}\,kR\cos\theta)^{-2/3}\sin^4\theta\Big] \tag{7.4}$$

$$\bar{g}_z = G(s)\Big[v(\xi)\Big(\cos^2\theta - \frac{j}{ks}\cos 2\theta\Big) + \frac{j}{ks}\,u(\xi)\sin^2\theta\Big]. \tag{7.5}$$

This solution gives an accurate numerical result (within several percent in magnitude and less than 5° in phase) provided that $kR \geq 10$ and the slot separation is greater than two wavelengths.

8. CONCLUDING REMARKS

Based on extensive numerical data, we conclude that Y_{12} (including Y_{11} as a special case) can be best calculated by

(i) Hughes modal solution if $kR \leq 5$ and z_0 is less than the axial dimension of the slot,

(ii) UI modal solution if $kR \leq 5$ and z_0 is greater than the axial dimension of the slot, and

(iii) UI asymptotic solution if $kR \geq 5$ for all slot separations.

If several percents of error are acceptable, the approximate solution can be used if $kR \geq 10$ and the slot separation is greater than two wavelengths.

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[15] S. W. Lee and S. Safavi-Naini, "Asymptotic solution of surface field due to a magnetic dipole on a cylinder," University of Illinois at Urbana-Champaign, Electromagnetics Laboratory Report No. 76-11, 1976.

APPENDIX A: NUMERICAL RESULTS

By using the formulas of Y_{12} presented in the text, we have analyzed the following 6 slots:

Slot	Type	Dimension	Suggested by
A	Circumf.	0.9" x 0.4" (f = 9 GHz)	Aerospace Hughes
B	Circumf.	$0.5\lambda \times 0.01\lambda$ (R in inch)	Hansen
C	Axial	0.4" x 0.9" (f = 9 GHz)	Aerospace Hughes
D	Circumf.	$0.5\lambda \times 0.01\lambda$ (R in λ)	Hansen
E	Circumf.	$0.5\lambda \times 0.2\lambda$	Hansen
F	Axial	$0.5\lambda \times 0.2\lambda$	

In all tables, Y_{12} is listed in (db, phase in degree) format where db = 20 $\log_{10}$ (Y_{12} in mho). In all figures, the normalized phase of Y_{12} is equal to $\mathrm{Arg}(Y_{12} \exp jks_0)$.

DATA SET A OF MUTUAL ADMITTANCE

(1) The mutual admittance Y_{12} between two circumferential slots on an infinitely long cylinder is calculated from the

* (Exact) Hughes modal solution
* (Exact) UI modal solution
* UI asymptotic solution
* OSU asymptotic solution
* PINY asymptotic solution.

The parameters are

* Frequency: $f = 9$ GHz, $k = 4.7878$ (inch)$^{-1}$, $\lambda = 1.3123''$
* Cylinder: $R = 1.991''$ unless specified otherwise
* Slot A: Circumferential

  $a = 0.9'' = 0.6858\lambda$

  $b = 0.4'' = 0.3048\lambda$

  $|Y_{11}| = 1.70747 \times 10^{-3}$ mho $= -55.35$ db

  $Y_g = 1.8155 \times 10^{-3}$ mho

* Center-to-center distance between two slots is $(R\phi_0, z_0)$.

(2) Y_{12} is listed in (db value, phase in degree), where

db value = 20 $\log_{10}$ ($|Y_{12}|$ in mho).

(3) Data are presented in

TABLE A-1: $\phi_0 = 0$ and various z_0

A-2: $z_0 = 2''$ and various ϕ_0

A-3: $z_0 = 0$ and various ϕ_0

A-4: $\phi_0 = 0$ and various z_0.

Figure A-1: Mutual admittance Y_{12} between two circumferential slots as a function of ϕ_0.

A-2 Mutual admittance Y_{12} between two circumferential slots as a function of z_0.

A-3: $|Y_{12}|$ on a cylinder (UI modal solution) and that on a plane as a function of z_0.

A-4: Y_{12} on a cylinder as a function of the radius R of the cylinder.

TABLE A-1

Y_{12} OF SLOT A FOR $\phi_o = 0$

	Modal		Asymptotic			Exact Planar
z_o	Hughes	UI	UI	OSU	PINY	R=∞
0.5"	-62.62 db -72^o	-62.62 -72^o	-62.54 -72^o	-64.22 -43^o	-61.7 -68^o	-63.69 -67^o
2"	-71.87 -117^o	-71.78 -117^o	-71.66 -116^o	-73.67 -100^o	-70.96 -118^o	-73.53 106^o
8"	-82.3 33^o	-81.84 34^o	-81.83 37^o	-85.46 55^o	-80.80 34^o	-85.4 54^o
16"		-86.48 -4^o	-86.6 -1^o	-91.41 20^o	-85.26 -4^o	-91.40 19^o
40"		-91.95 -115^o	-92.46 -110^o	-99.34 -83^o	-90.83 -112^o	-99.33 -83^o

TABLE A-2

Y_{12} OF SLOT A FOR z_o = 2"

ϕ_o	Modal		Asymptotic		
	Hughes	UI	UI	OSU	PINY
0°	-71.87 db -117°	-71.78 -117°	-71.66 -116°	-73.67 -100°	-70.96 -118°
30°	-77.60 175°	-77.42 175°	-77.69 177°	-79.25 170°	-76.6 172°
60°	-89.98 -4°	-90.00 -3°	-90.17 -1°	-91.11 6°	-88.41 -10°
90°	-103.15 116°	-102.52 120°	-103.10 116°	-103.83 119°	-101.69 106°

TABLE A-3

Y_{12} OF SLOT A FOR z_o = 0

ϕ	Modal	Asymptotic		
	Hughes	UI	OSU	PINY
30°	-81.33 db -77°	-81.34 -75°	-89.72 -62°	-83.14 -60°
40°	-89.87 168°	-90.02 170°	-98.66 174°	-91.11 -180°
50°	-96.37 58°	-96.72 61°	-105.95 58°	-97.43 69°
60°	-101.97 -49°	-102.48 -47°	-112.59 -55°	-102.93 -39°

TABLE A-4

UI SOLUTIONS OF Y_{12} OF SLOT A FOR $\phi_o = 0$

z_o	Modal	Asymptotic	z_o	Modal	Asymptotic
0.5"	-62.62 db -72°	-62.54 -72°	11"	-84.06 -70°	-84.06 -68°
1"	-66.82 155°	-66.71 155°	12"	-84.61 15°	-84.65 18°
2"	-71.78 -117°	-71.66 -116°	13"	-85.12 100°	-85.20 103°
3"	-74.78 -31°	-74.67 -30°	14"	-85.63 -175°	-85.70 -172°
4"	-76.89 54°	-76.89 54°	15"	-86.09 -90°	-86.17 -86°
5"	-78.51 139°	-78.44 141°	16"	-86.48 -4°	-86.60 -1°
6"	-79.85 -136	-79.77 -134	17"	-86.85 81	-87.01 84
7"	-80.94 -51°	-80.88 -49°	18"	-87.24 166°	-87.38 170°
8"	-81.84 34°	-81.83 37°	20"	-87.91 -24°	-88.08 -19°
9"	-82.65 119°	-82.66 122°	30"	-90.33 110°	-90.68 115°
10"	-83.40 -156°	-83.40 -153°	40"	-91.95 -115°	-92.46 -110°

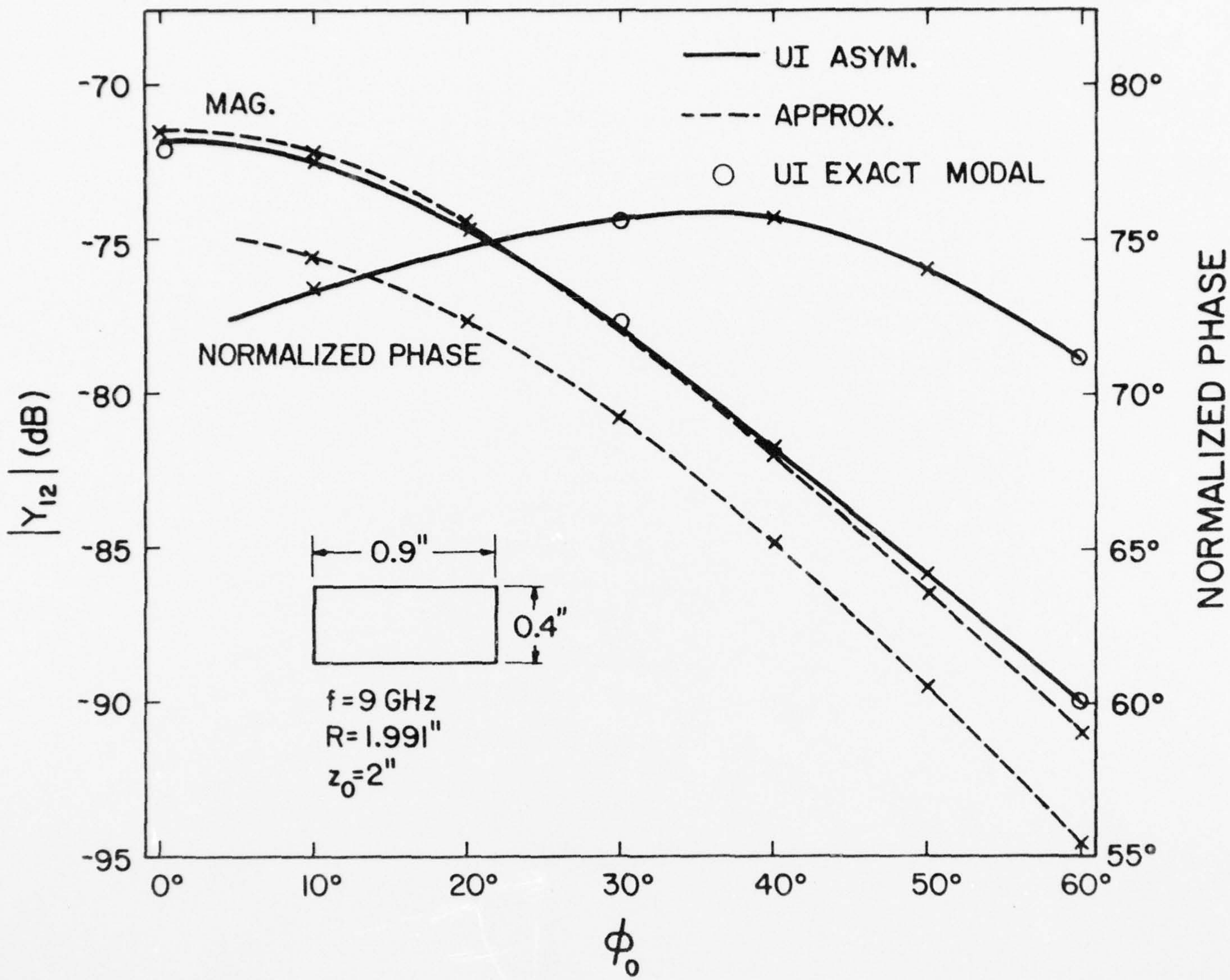

Figure A-1. Mutual admittance Y_{12} between two circumferential slots as a function ϕ_0.

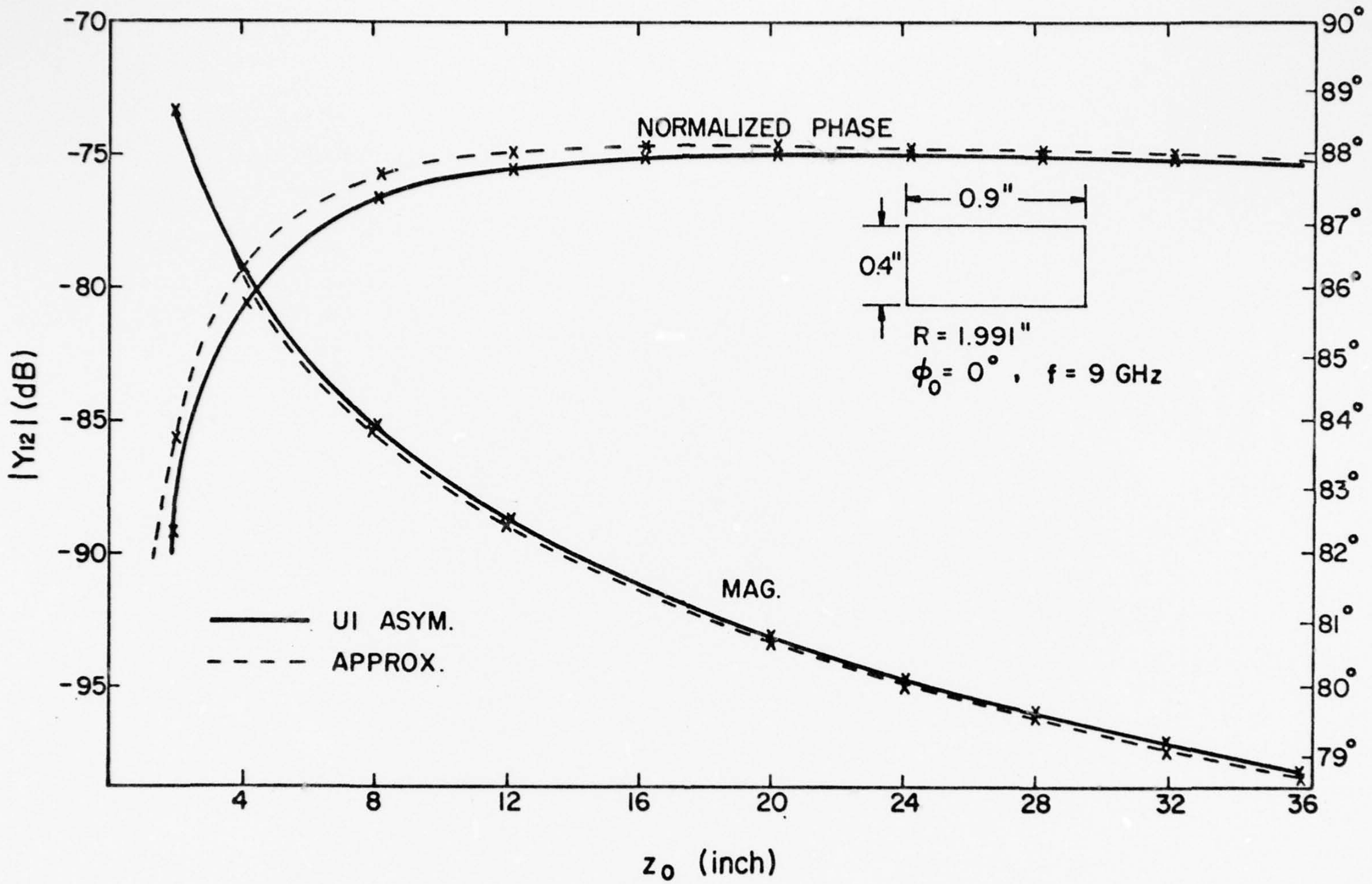

Figure A-2. Mutual admittance Y_{12} between two circumferential slots as a function of z_0.

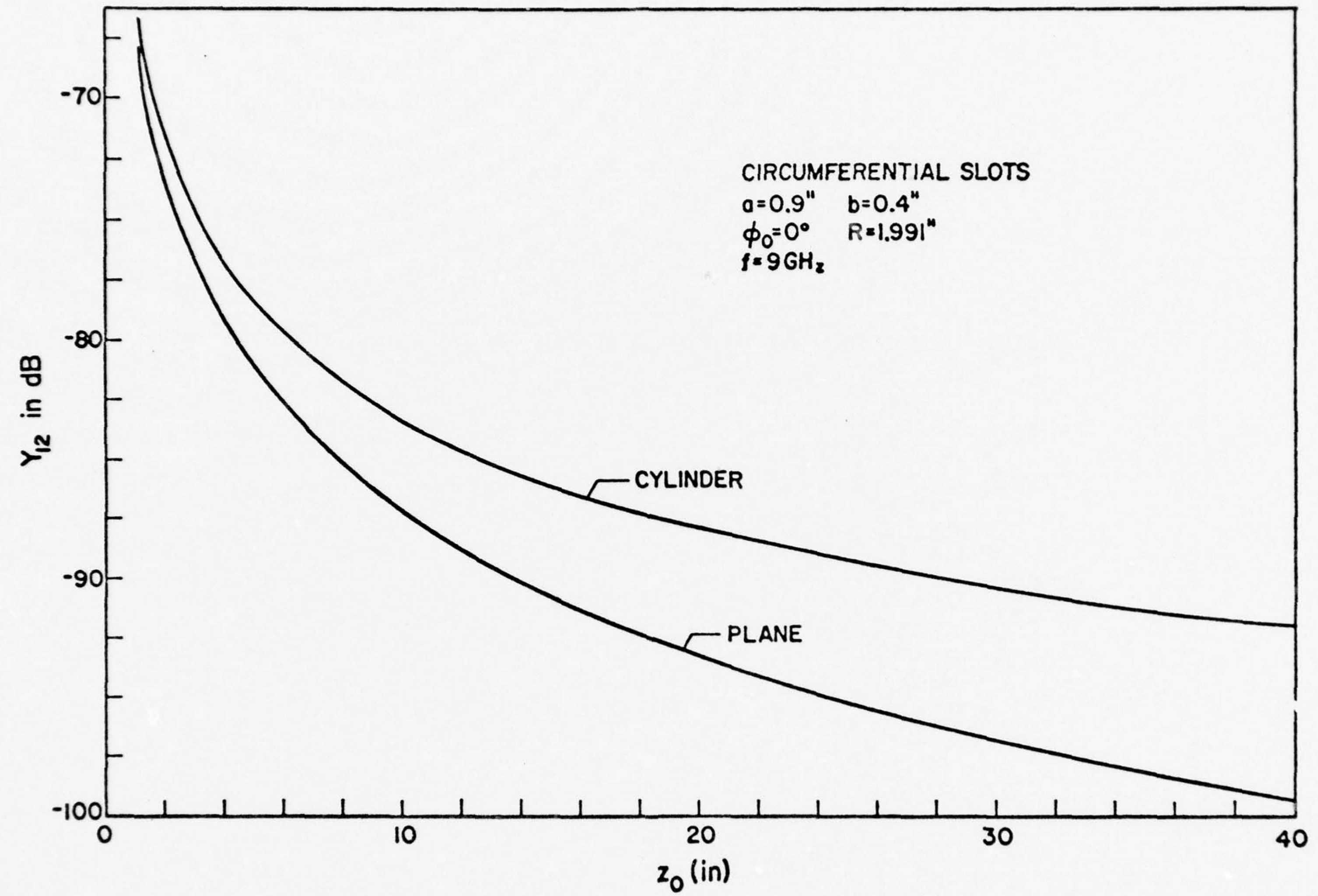

Figure A-3: $|Y_{12}|$ on a cylinder (UI modal solution) and that on a plane as a function of z_0.

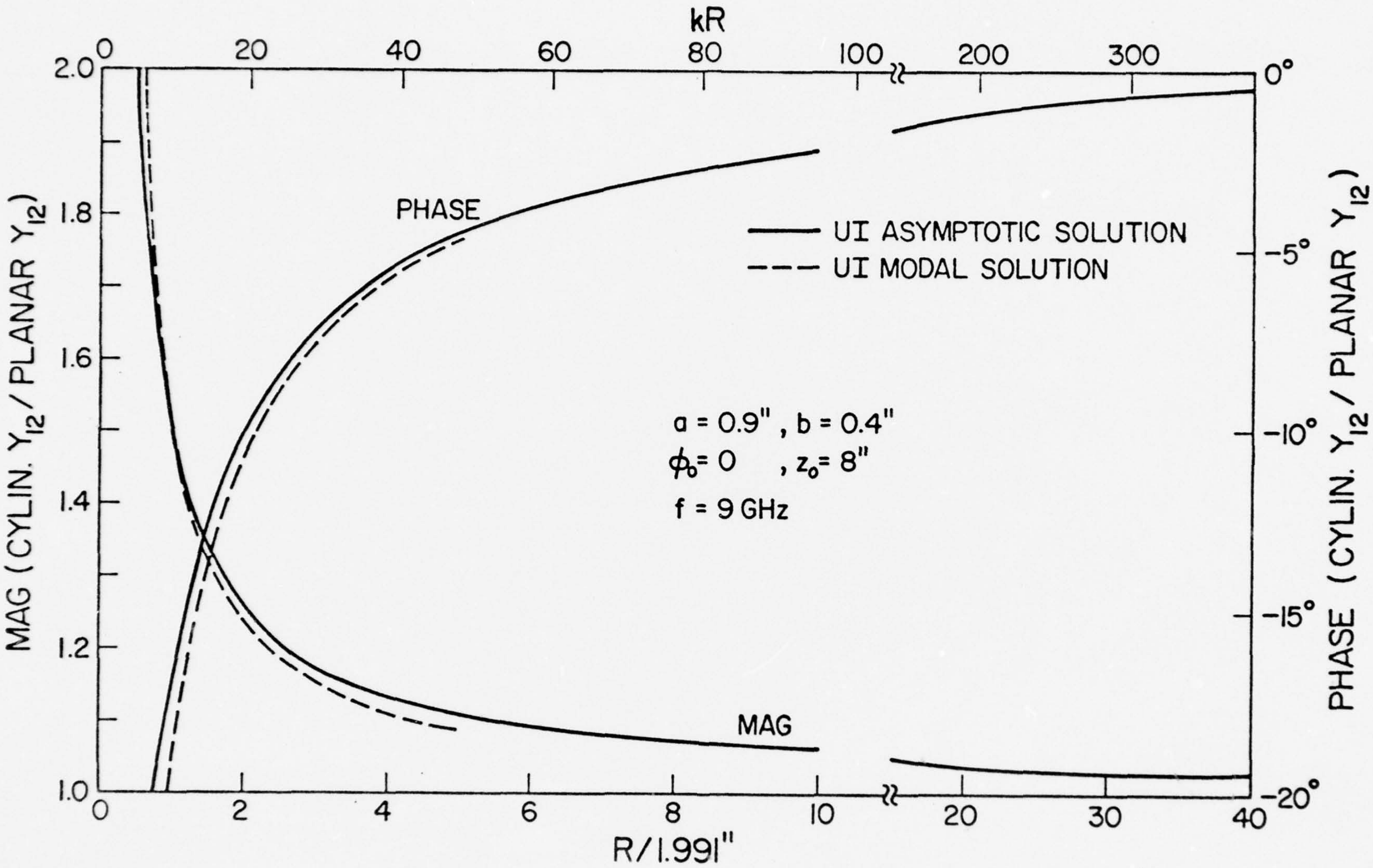

Figure A-4: Y_{12} on a cylinder as a function of the radius R of the cylinder. Y_{12} is normalized by Y_{12} on a plane which is 5.37×10^{-5} exp(j53.55°) mho.

DATA SET B OF MUTUAL ADMITTANCE

(1) The mutual admittance Y_{12} between two circumferential slots on an infinitely long cylinder is calculated from the

* (Exact) UI modal solution
* UI asymptotic solution.

The parameters are

* Frequency: $f = 9$ GHz, $k = 4.787787$ (inch)$^{-1}$, $\lambda = 1.3123''$
* Cylinder: $R = 1.991''$, $3.777''$, $6''$
* Slot B: Circumferential

  $a = 0.656168'' = 0.50\lambda$

  $b = 0.013123'' = 0.01\lambda$
* Center-to-center distance between two slots is $(R\phi_0, z_0)$.

(2) Y_{12} is listed in (db value, phase in degree), where

$$\text{db value} = 20 \log_{10} (|Y_{12}| \text{ in mho}).$$

(3) Data are presented in

TABLE B-1: $\phi_0 = 0$ and various z_0

B-2: $z_0 = 2''$ and various ϕ_0

B-3: $z_0 = 8''$ and various ϕ_0.

B-4: Comparison of Hughes and UI solutions

TABLE B-1

UI SOLUTIONS OF Y_{12} OF SLOT B FOR $\phi_o = 0$

	R = 1.991"		R = 3.777"		R = 6"		Exact Planar R=∞
z_o	Modal	Asymp	Modal	Asymp	Modal	Asymp	
0.5"	-92.00 db -79°	-92.03 -78°	-92.48 -77°	-92.52 -78°	-92.70 -76°	-92.74 -76°	-93.11 -74°
1"	-96.31 152°	-96.28 153°	-96.97 156°	-96.92 159°	-97.24 157°	-97.19 157°	-97.61 155°
2"	-101.33 -117°	-101.32 -116°	-102.20 -113°	-102.17 -113°	-102.56 -111°	-102.54 -111°	-103.20 -109°
4"	-106.50 54°	-106.51 56°	-107.70 60°	-107.66 61°	-108.23 63°	-108.77 63°	-109.10 67°
8"	-111.48 36°	-111.56 37°	-113.13 42°	-113.11 43°	-113.85 46°	-113.81 46°	-115.08 53°
16"	-116.13 -4°	-116.35 -1°	-118.37 5°	-118.38 6°	-119.36 10°	-119.33 10°	-121.10 20°

TABLE B-2

UI SOLUTIONS OF Y_{12} OF SLOT B FOR z_o = 2"

	R = 1.991"		R = 3.777"		R = 6.0"	
ϕ_o	Modal	Asymp	Modal	Asymp	Modal	Asymp
10°	-102.01 db -125°	-102.04 -125°	-104.18 -140°	-104.22 -140°	-106.89 -177°	-106.94 -177°
20°	-103.94 -149°	-104.11 -148°	-109.18 142°	-109.36 143°	-115.80 11°	-115.93 12°
30°	-106.86 172°	-107.20 173°	-115.53 27°	-115.75 28°	-124.77 140°	-124.95 141°
45°	-112.51 92°	-112.98 93°	-125.07 169°	-125.40 170°	-136.67 106°	-136.82 105°
60°	-119.01 -11°	-119.28 -9°	-134.48 -81°	-134.38 -77°	-148.07 51°	-147.24 44°
90°	-131.40 110°	-131.83 106°	-148.22 132°	-150.57 113°	-155.92 -170°	-165.47 -102°

TABLE B-3

UI SOLUTIONS OF Y_{12} OF SLOT B FOR z_o = 8"

	R = 1.991"		R = 3.777"		R = 6.0"	
ϕ_o	Modal	Asymp	Modal	Asymp	Modal	Asymp
10°	-111.63 db 32°	-111.74 34°	-113.45 34°	-113.47 34°	-114.44 26°	-114.46 26°
20°	-112.08 24°	-112.29 26°	-114.40 9°	-114.54 9°	-116.18 -34°	-116.32 -34°
30°	-112.83 11°	-113.18 13°	-115.94 -32°	-116.26 -32°	-118.94 -130°	-119.21 -129°
45°	-114.41 -17°	-115.12 -16°	-119.29 -122°	-119.82 -121°	-124.43 27°	-124.85 29°
60°	-116.70 -56°	-117.70 -55°	-123.69 118°	-124.22 121°	-131.31 127°	-131.37 130°
90°	-122.98 -161°	-124.10 159°	-134.62 169°	-134.27 172°	-146.21 -132°	-145.33 146°

TABLE B-4

COMPARISON OF HUGHES AND UI SOLUTIONS

		R = 1.991"			R = 3.777"			R = 6"		
ϕ_o	z_o	Hughes Modal	UI		Hughes Modal	UI		Hughes Modal	UI	
			Modal	Asymp		Modal	Asymp		Modal	Asymp
0°	0.5"	-92.3 db -79°	-92 -79°	-92.03 -78°	-92.83 -77°	-92.48 -77°	-92.52 -78°	-92.87 -76°	-92.70 -76°	-92.74 -76°
	1"	-96.5 153°	-96.31 152°	-96.28 153°	-97.18 157°	-96.97 156°	-96.92 159°	-97.34 156°	-97.24 157°	-97.19 157°
	8"	-112.02 33°	-111.5 36°	-111.56 37°	-113.65 40°	-113.13 42°	-113.11 43°	-114.42 44°	-113.85 46°	-113.81 46°
	16"	-117.08 -6°	-116.13 -4°	-116.35 -1°	-119.27 3°	-118.37 5°	-118.38 6°		-119.36 10°	-119.33 10°
45°	2"	-112.73 91°	-112.51 92°	-112.98 93°	-125.43 168°	-125.07 169°	-125.40 170°	-137.17 104°	-136.7 106°	-136.82 105°

DATA SET C OF MUTUAL ADMITTANCE

(1) The mutual admittance Y_{12} between two <u>axial</u> slots on an infinitely long cylinder is calculated from the

* (Exact) UI modal solution
* UI asymptotic solution

The parameters are

* Frequency: $f = 9$ GHz, $k = 4.7877$ (inch)$^{-1}$, $\lambda = 1.3123''$
* Cylinder: $R = 1.991''$, and other values
* Slot C: Axial

$a = 0.4'' = 0.3048\lambda$

$b = 0.9'' = 0.6858\lambda$

* Center-to-center distance between two slots is $(R\phi_0, z_0)$.

(2) Y_{12} is listed in (db value, phase in degree), where db value = $20 \log_{10}$ ($|Y_{12}|$ in mho)

(3) Data are presented in

TABLE C-1: $\phi_0 = 0$, $R = 1.991''$, and various z_0.

C-2: $z_0 = 1.5''$, $R = 1.991''$, and various ϕ_0.

C-3: $\phi_0 = 0$, $z_0 = 8''$, and various R.

Figure C-1: $|Y_{12}|$ on a cylinder (UI modal solution) and that on a plane as a function of z_0.

TABLE C-1

Y_{12} OF SLOT C FOR $\phi_o = 0^o$

z_o	Modal	Asymp	z_o	Modal	Asymp
1"	-77.38db -59°	-77.28 -59°	12"	-123.86 134°	-123.55 130°
2"	-92.00 8°	-91.86 6°	14"	-127.50 -51°	-126.23 -59°
3"	-99.48 89°	-99.25 86°	16"	-128.96 115°	-128.55 112°
4"	-104.68 172°	-104.36 170°	18"	-131.64 -68°	=130.60 -76°
5"	-108.88 -103°	-108.28 -106°	20"	-133.39 102°	-132.43 95°
6"	-111.94 -17°	-111.48 -21°	24"	-136.07 81°	-135.59 77°
7"	-114.61 68°	-114.17 64°	28"	-138.79 72°	-138.27 60°
8"	-116.93 151°	-116.5 149°	32"	-141.24 59°	-140.59 42°
9"	-119.28 -122°	-118.55 -126°	36"	-143.68 39°	-142.63 25°

TABLE C-2

Y_{12} OF SLOT C FOR $z_0 = 1.5''$

ϕ_0	Modal	Asymptotic
0^o	-86.58 db 151^o	-86.31 149^o
30^o	-86.41 -26^o	-85.15 -38^o
60^o	-87.43 84^o	-85.77 72^o
90^o	-93.02 169^o	-91.04 156^o

TABLE C-3

Y_{12} OF SLOT C FOR $\phi_o = 0$ and $z_o = 8''$

R	Modal	Asymptotic
0.995"	-118.07 db 150°	-116.55 148°
1.991"	-116.93 151°	-116.50 149°
3.982"	-116.91 150°	-116.47 149°
5.973"	-116.90 154°	-116.46 149°
7.964"	-116.89 154°	-116.45 149°
11.946"	-116.84 153°	-116.45 149°
15.928"	-116.82 153°	-116.45 149°
19.910"		-116.44 149°

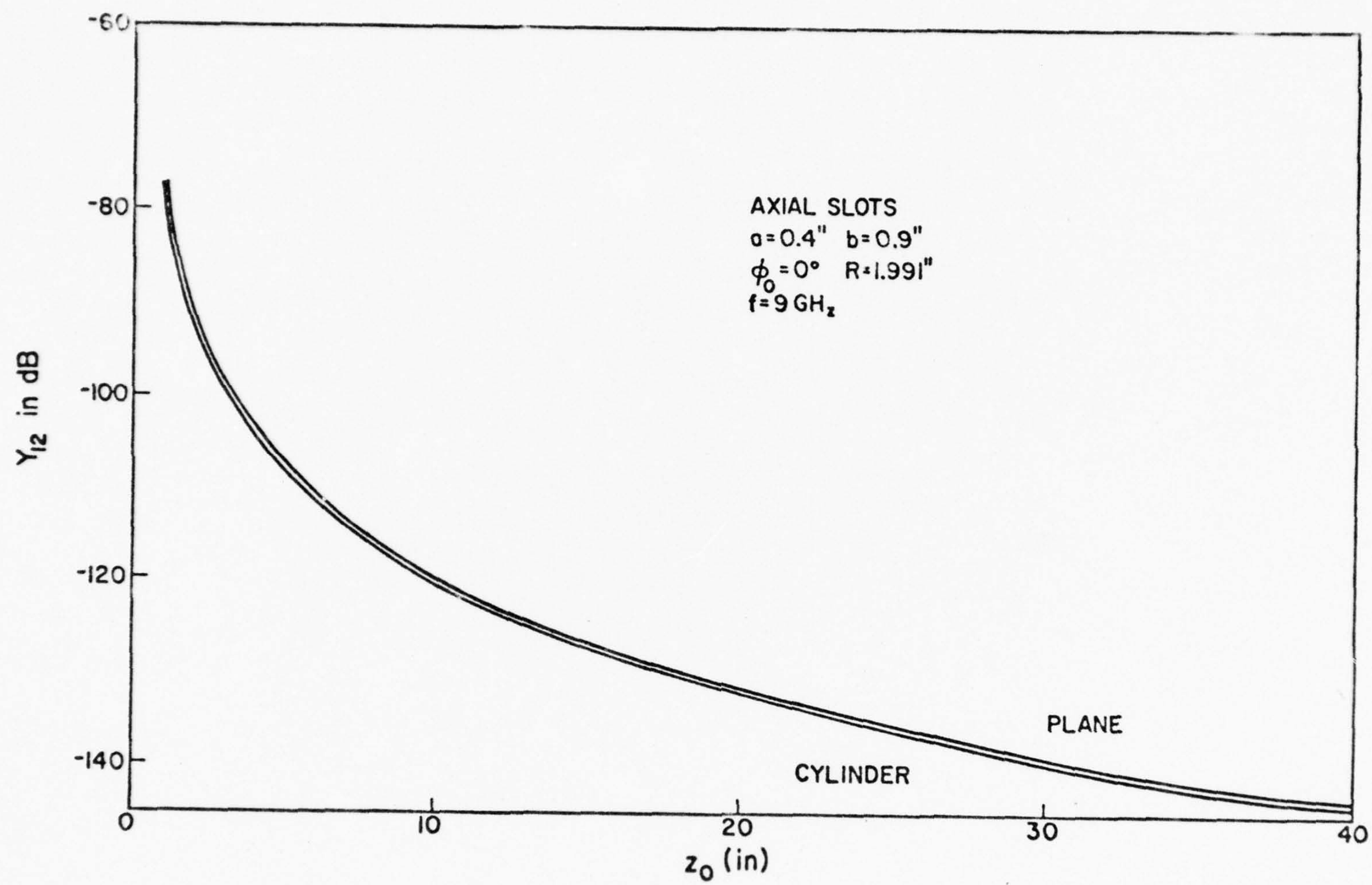

Figure C-1: $|Y_{12}|$ on a cylinder (UI modal solution) and that on a plane as a function of z_0.

DATA SET D OF MUTUAL ADMITTANCE

(1) The mutual admittance Y_{12} between two circumferential slots on an infinitely long cylinder from the

* (Exact) UI modal solution
* UI asymtotic solution

The parameters are

* Cylinder: $R = 1\lambda, 2\lambda, 4\lambda, 10\lambda, \infty$ (planar)
* Slot D: Circumferential

  $a = 0.5\lambda$

  $b = 0.01\lambda$
* Center-to-center distance between two slots is $(R\phi_o, z_o)$

(2) Y_{12} is listed in (db value, phase in degree), where

db value = $20 \log_{10}$ ($|Y_{12}|$ in mho)

(3) Data are presented in

TABLE D-1: $\phi_o = 0$, $R = 2\lambda$ and various z_o

D-2: $\phi_o = 0$ and various R and z_o

D-3: $\phi_o = 0$ and various R and z_o

D-4: $z_o = 0$ and various R and ϕ_o

D-5: $z_o = 1\lambda$ and various R and ϕ_o

D-6: $z_o = 5\lambda$ and various R and ϕ_o

TABLE D-1

UI SOLUTIONS OF Y_{12} OF SLOT D FOR $\phi_o = 0$ and $R = 2\lambda$

z_o	Modal	Asymptotic
1λ	-98.60 db 71^o	-98.56 71^o
2λ	-103.87 74^o	-103.84 75^o
3λ	-106.98 75^o	-106.96 75^o
4λ	-109.17 74^o	-109.16 75^o
5λ	-110.84 73^o	-110.85 75^o
6λ	-112.19 73^o	-112.21 74^o
7λ	-113.32 72^o	-113.35 74^o
8λ	-114.28 72^o	-114.33 73^o
9λ	-115.12 71^o	-115.18 73^o
10λ		-115.94 72^o

TABLE D-2

UI ASYMPTOTIC SOLUTIONS OF Y_{12} OF SLOT D FOR $\phi_o = 0$

z_o	R = 1λ	R = 2λ	R = 4λ	R = 10λ	Planar (R = ∞)
0	81.51 db 90^o	81.51 90^o	81.51 90^o	81.51 90^o	
1 λ	-97.49 67^o	-98.56 71^o	-99.15 74^o	-99.51 76^o	-99.76 77^o
2 λ	-102.39 69^o	-103.84 75^o	-104.63 79^o	-105.13 81^o	-105.47 83^o
3 λ	-105.26 69^o	-106.96 75^o	-107.92 80^o	-108.52 83^o	-108.93 86^o
4 λ	-107.25 68^o	-109.16 75^o	-110.25 80^o	-110.94 84^o	-111.40 87^o
5 λ	-108.76 67^o	-110.85 75^o	-112.05 80^o	-112.81 84^o	-113.33 87^o
6 λ	-109.97 67^o	-112.21 74^o	-113.51 80^o	-114.34 84^o	-114.91 88^o
7 λ	-110.98 66^o	-113.35 74^o	-114.74 80^o	-115.63 84^o	-116.25 88^o
8 λ	-111.85 65^o	-114.33 73^o	-115.80 79^o	-116.75 84^o	-117.40 88^o
9 λ	-112.60 65^o	-115.18 73^o	-116.72 79^o	-117.73 84^o	-118.43 89^o
10 λ	-113.27 64^o	-115.94 72^o	-117.55 79^o	-118.61 84^o	-119.34 89^o

TABLE D-3

UI ASYMPTOTIC SOLUTIONS OF Y_{12} OF SLOT D FOR $\phi_o = 0$

z_o	R = 1λ	R = 2λ	R = 4λ	R = 10λ	Planar (R = ∞)
0.5λ	-93.01 db -119°	-93.83 -116°	-94.27 -115°	-94.55 -114°	-94.74 -113°
1.5λ	-100.34 -111°	-101.62 -106°	-102.32 -103°	-102.76 -100°	-103.05 -99°
2.5λ	-103.98 -111°	-105.56 -104°	-106.44 -100°	-106.99 -98°	-107.37 -95°

TABLE D-4

UI ASYMPTOTIC SOLUTIONS OF Y_{12} OF SLOT D FOR $z_o = 0$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
10^o	-7.62 db 90^o	-43.02 90^o	-106.09 -59^o	-124.64 -92^o
20^o	-43.02 90^o	-107.10 -62^o	-121.97 29^o	-140.01 -20^o
30^o	-98.90 19^o	-117.45 155.34^o	-131.85 122^o	-150.99 52^o
45^o	-112.59 -106^o	-128.38 -52^o	-143.53 84^o	-164.60 165^o
60^o	-121.31 143^o	-137.31 102^o		-175.95 -74^o

TABLE D-5

UI ASYMPTOTIC SOLUTIONS OF Y_{12} OF SLOT D FOR $z_o = 1\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
10^o	-98.12 db 62^o	-100.56 54^o	-105.46 9^o	-120.20 107^o
20^o	-99.93 48^o	-105.62 4^o	-116.60 -160^o	-136.65 -115^o
30^o	-102.69 26^o	-111.94 -75^o	-126.33 -18^o	-148.10 -17^o
45^o	-107.99 -24^o	-121.35 134^o	-138.27 -21^o	-162.12 111^o
60^o	113.88 -89^o	-129.91 -40^o	-148.52 -40^o	-174.13 -123^o

TABLE D-6

UI ASYMPTOTIC SOLUTIONS OF Y_{12} OF SLOT D FOR $z_o = 5\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
10^o	-108.90 db 66^o	-111.13 70^o	-112.73 61^o	-115.49 -25^o
20^o	-109.31 61^o	-111.98 54^o	-114.67 6^o	-121.98 39^o
30^o	-109.98 53^o	-113.34 29^o	-117.66 -83^o	-129.87 -24^o
45^o	-111.47 37^o	-116.23 -26^o	-123.42 91^o	-141.86 -72^o
60^o	-113.50 14^o	-119.88 -99^o	-130.02 -145^o	-153.21 159^o

DATA SET E OF MUTUAL ADMITTANCE

(1) The mutual admittance Y_{12} between two circumferential slots on an infinitely long cylinder is calculated from the

* UI asymptotic solution

The parameters are

*Cylinder: $R = 1\lambda, 2\lambda, 4\lambda, 10\lambda$

*Slot E: Circumferential

$a = 0.5\lambda$

$b = 0.2\lambda$

*Center-to-center distance between two slots is $(R\phi_o, z_o)$

(2) Y_{12} is listed in (db value, phase in degree), where

db value = $20 \log_{10} (|Y_{12}|$ in mho)

(3) Data are presented in

TABLE E-1: $z_o = o$, various ϕ_o and R

E-2: $z_o = 0.5\lambda$, various ϕ_o and R

E-3: $z_o = 1\lambda$, various ϕ_o and R

E-4: $z_o = 2\lambda$, various ϕ_o and R

E-5: $z_o = 4\lambda$, various ϕ_o and R

E-6: $z_o = 8\lambda$, various ϕ_o and R

E-7: Comparison of UI asymptotic and UI modal solutions

E-8: Comparison of UI asymptotic and UI modal solutions

Figure E-1: Mutual admittance Y_{12} between two circumferential slots as a function of ϕ_0.

E-2: Mutual admittance Y_{12} between two circumferential slots as a function of ϕ_0.

E-3: Mutual admittance Y_{12} between two circumferential slots as a function of ϕ_0.

E-4: Mutual admittance Y_{12} between two circumferential slots as a function of z_0.

E-5: Mutual admittance Y_{12} between two circumferential slots as a function of z_0.

TABLE E-1

UI ASYMPTOTIC SOLUTIONS OF Y_{12} OF SLOT E FOR $z_o = 0$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
30^o	-73.94 7^o	-91.47 153^o	-105.83 121^o	-124.96 52^o
45^o	-86.67 -110^o	-102.35 -54^o	-117.50 83^o	-138.57 165^o
60^o	-95.31 140^o	-111.28 101^o	-127.44 49^o	-149.93 77^o

TABLE E-2

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 0.5\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
0^o	-67.67 db -117^o	-68.46 -114^o	-68.89 -112^o	-69.16 -111^o
10^o	-69.00 -122^o	-72.97 -132^o	-81.72 170^o	-98.38 -146^o
20^o	-72.67 -137^o	-82.21 164^o	-95.39 -39^o	-113.59 -49^o
30^o	-77.77 -165^o	-90.67 64^o	-105.02 75^o	-124.52 32^o
45^o	-85.89 130^o	-100.98 -116^o	-116.60 50^o	-138.17 150^o
60^0	-93.37 47^o	-109.75 51^o	-126.60 20^o	-149.69 -90^o

TABLE E-3

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 1\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
0^o	-72.28 db 68^o	-73.34 73^o	-73.92 76^o	-74.28 78^o
10^o	-72.91 64^o	-75.33 55^o	-80.15 9^o	-94.52 105^o
20^o	-74.71 49^o	-80.31 3^o	-91.02 -161^o	-110.75 -116^o
30^o	-77.44 26^o	-86.49 -76^o	-100.56 -20^o	-122.14 -18^o
45^o	-82.65 -24^o	-95.69 132^o	-112.38 -22^o	-136.13 111^o
60^o	-88.42 -90^o	-104.13 -42^o	-122.57 -41^o	-148.13 -124^o

TABLE E-4

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 2\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
0^o	-77.24 db 70^o	-78.67 76^o	-79.46 80^o	-79.96 82^o
10^o	-77.52 67^o	-79.44 65^o	-81.77 37^o	-89.55 -148^o
20^o	-78.37 57^o	-81.60 31^o	-87.38 -79^o	-103.31 76^o
30^o	-79.73 42^o	-84.81 -21^o	-94.18 112^o	-114.68 -144^o
45^o	-82.59 9^o	-90.76 -130^o	-104.37 162^o	-128.88 15^o
60^o	-86.17 -35^o	-97.24 94^o	-113.88 177^o	-141.23 153^o

TABLE E-5

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 4\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
0^o	-82.10 db 68^o	-84.01 75^o	-85.10 81^o	-85.78 84^o
10^o	-82.26 67^o	-82.26 67^o	-85.97 57^o	-89.37 -49^o
20^o	-82.73 61^o	-82.73 61^o	-88.41 -10^o	-97.35 -37^o
30^o	-83.51 52^o	-83.51 52^o	-92.03 -116^o	-106.24 -149^o
45^o	-85.21 32^o	-85.21 32^o	-98.74 26^o	-118.99 -108^o
60^o	-87.48 5^o	-87.48 5^o	-106.08 117^o	-130.69 -58^o

TABLE E-6

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 8\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
0^o	-86.70 db 66^o	-89.18 73^o	-90.65 80^o	-91.60 85^o
10^o	-86.81 64^o	-89.38 70^o	-91.06 67^o	-92.96 14^o
20^o	-87.12 61^o	-89.97 60^o	-92.26 31^o	-96.69 171^o
30^o	-87.63 56^o	-90.93 43^o	-94.18 -28^o	-101.98 -140^o
45^o	-88.77 44^o	-93.03 5^o	-98.14 -156^o	-111.31 -35^o
60^o	-90.35 27^o	-95.78 -45^o	-102.98 34^o	-121.14 -47^o

TABLE E-7

COMPARISON OF UI ASYMPTOTIC AND UI MODAL SOLUTIONS

z_o	ϕ_o	R = 1λ		R = 2λ	
		Modal	Asym.	Modal	Asym
1λ	0^o	-72.54 db 67^o	-72.28 68^o	-73.64 73^o	-73.34 73^o
	10^o	-73.12 63^o	-79.91 64^o	-75.54 55^o	-75.33 55^o
	20^o	-74.78 48^o	-74.71 49^o	-80.33 3^o	-80.31 3^o
	30^o	-77.34 25^o	-77.44 26^o	-86.37 -77	-86.49 76^o
	45^o	-82.3 -26^o	-82.65 -24^o	-95.62 130^o	-95.69 132^o
	60^o	-88.05 -91^o	-88.42 -90^o	-103.77 -41^o	-104.13 -42^o

TABLE E-8

COMPARISON OF UI ASYMPTOTIC AND UI MODAL SOLUTIONS

z_o	ϕ_o	R = 1λ		R = 2λ		Planar (Exact)
		Modal	Asym.	Modal	Asym.	
0^o	0.5λ	-67.87 db -117^o	-67.67 -117^o	-68.69 -114^o	-68.46 -114^o	-69.35 -110^o
	1λ	-72.54 67^o	-72.28 68^o	-73.64 73^o	-73.34 73^o	-74.52 79^o
	2λ	-77.46 68^o	-77.24 70^o	-78.98 75^o	-78.67 76^o	-80.29 84^o
	4λ	-82.22 66^o	-82.10 68^o	-84.3 75^o	-84.01 75^o	-86.25 87^o
	8λ	-86.65 62^o	-86.7 66^o	-89.41 72^o	-89.18 73^o	-92.25 89^o

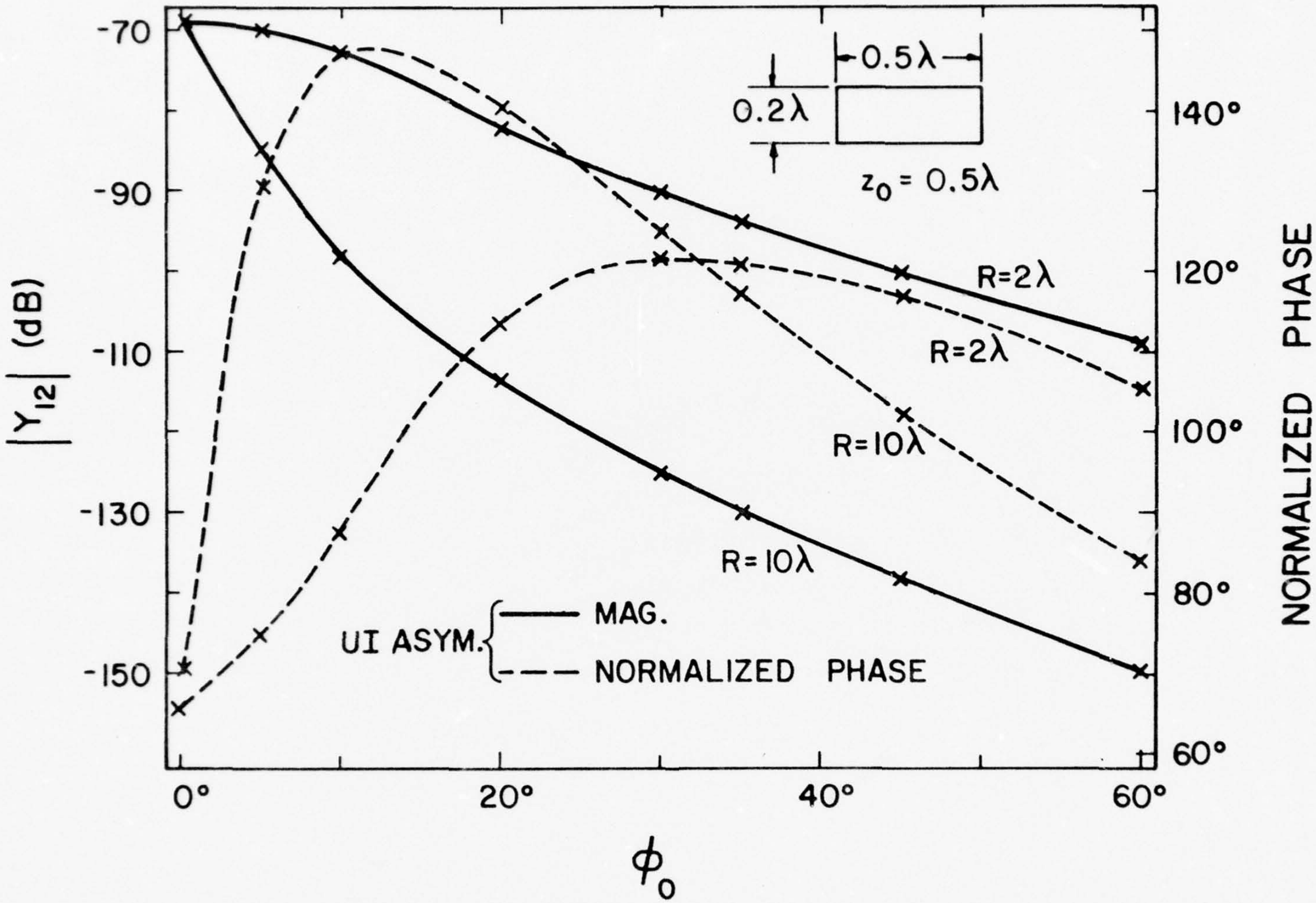

Figure E-1. Mutual admittance Y_{12} between two circumferential slots as a function of ϕ_0.

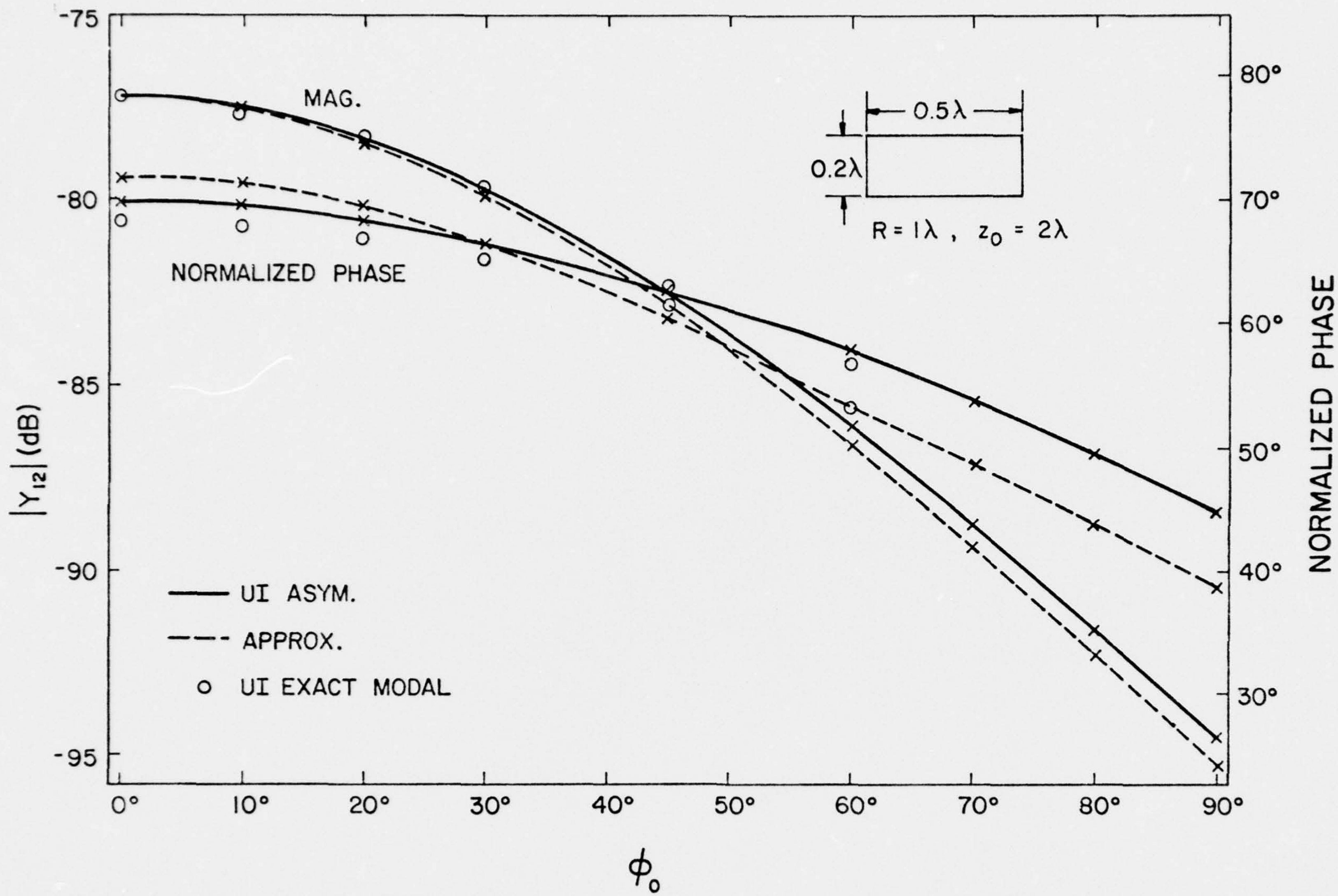

Figure E-2. Mutual admittance Y_{12} between two circumferential slots as a function of ϕ_0.

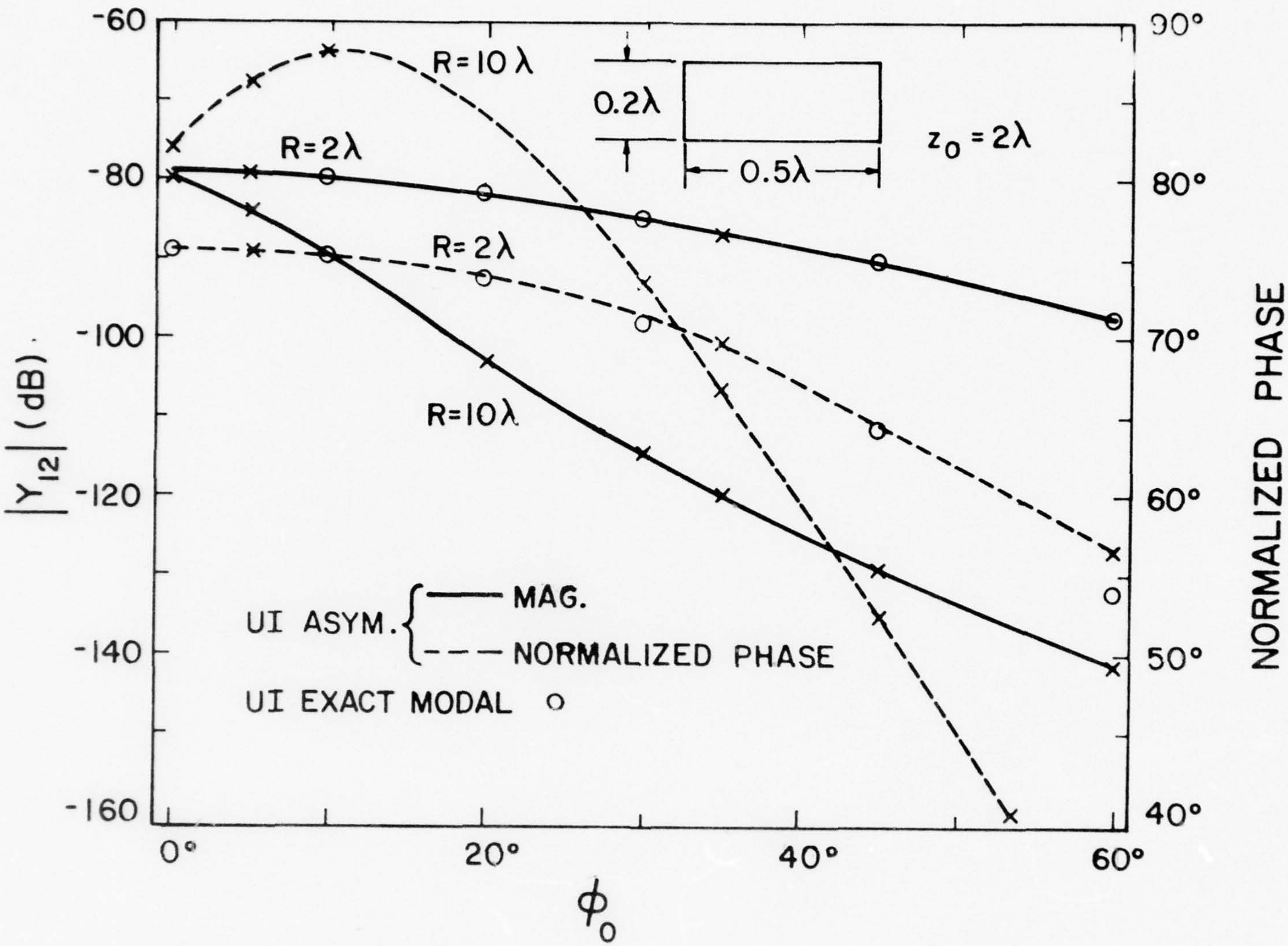

Figure E-3. Mutual admittance Y_{12} between two circumferential slots as a function of ϕ_0.

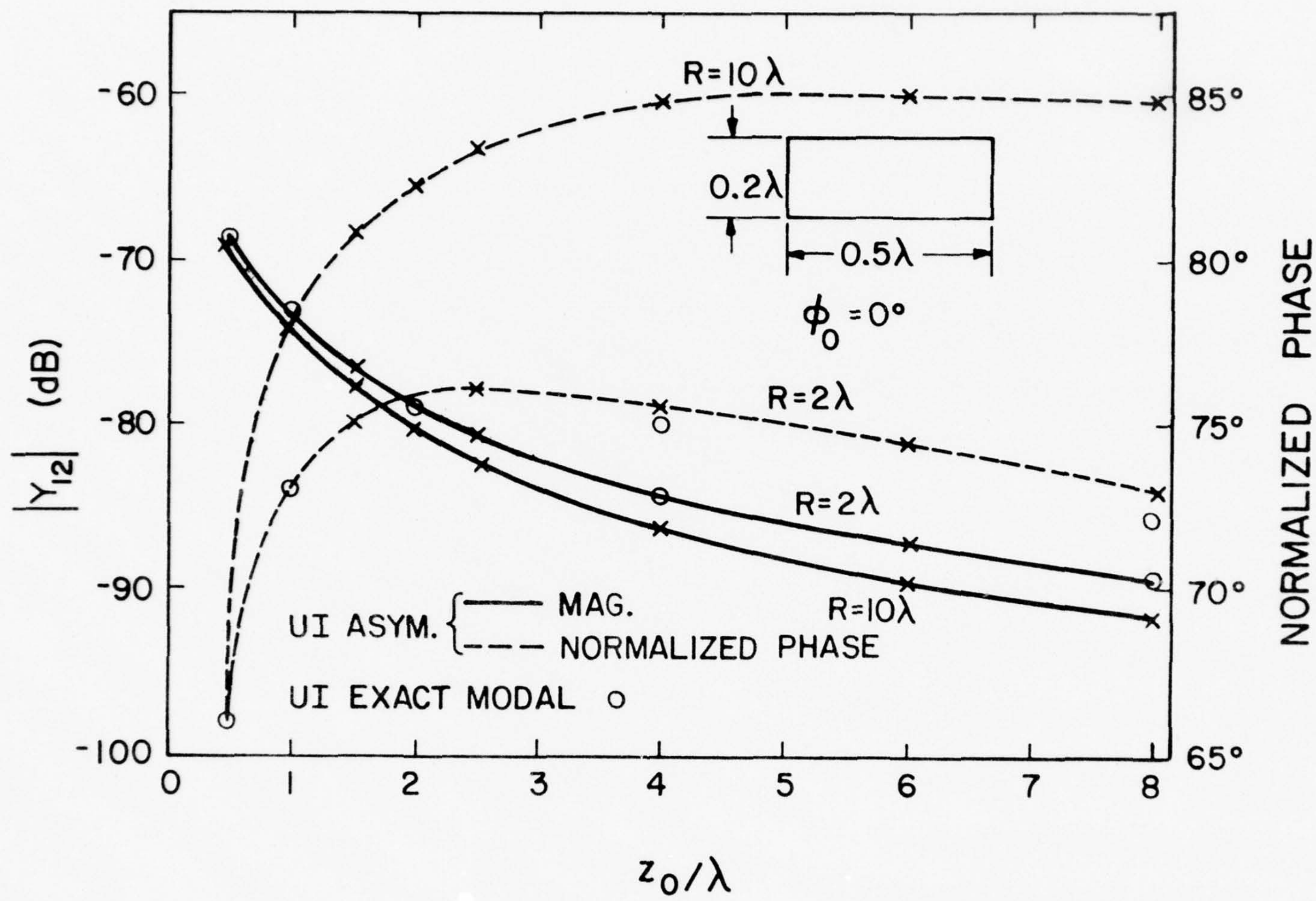

Figure E-4. Mutual admittance Y_{12} between two circumferential slots as a function of z_0.

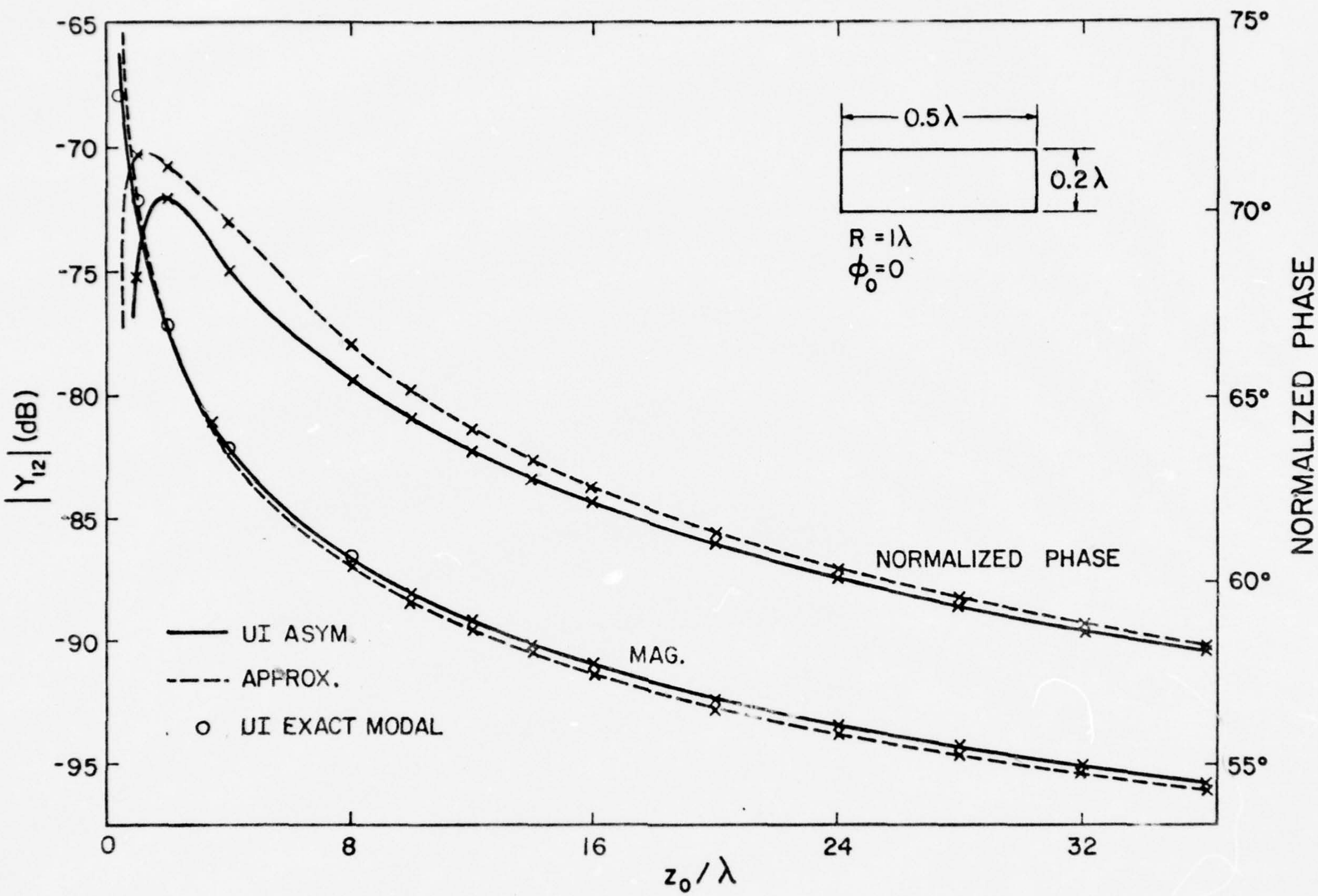

Figure L-5. Mutual admittance Y_{12} between two circumferential slots as a function of z_0.

DATA SET F OF MUTUAL ADMITTANCE

(1) The mutual admittance Y_{12} between two axial slots on an infinitely long cylinder is calculated from the

*UI asymptotic solution

The parameters are

*Cylinder: $R = 1\lambda,\ 2\lambda,\ 4\lambda,\ 10\lambda$

*Slot F: Axial

$a = 0.2\lambda$

$b = 0.5\lambda$

*Center-to-center distance between two slots is $(R\phi_o,\ z_o)$

(2) Y_{12} is listed in (db value, phase in degree), where

db value = $20 \log_{10} (|Y_{12}|$ in mho).

(3) Data are presented in

TABLE F-1: $z_o = 0$, various ϕ_o and R

F-2: $z_o = 0.5\lambda$, various ϕ_o and R

F-3: $z_o = 1\lambda$, various ϕ_o and R

F-4: $z_o = 2\lambda$, various ϕ_o and R

F-5: $z_o = 4\lambda$, various ϕ_o and R

F-6: $z_o = 8\lambda$, various ϕ_o and R

F-7: Comparison of UI asymptotic and UI modal solutions

F-8: Comparison of UI asymptotic and UI modal solutions

F-9: Comparison of asymptotic solutions

Figure F-1: Mutual admittance Y_{12} between two axial slots as a function of ϕ_0.

F-2: Mutual admittance Y_{12} between two axial slots as a function of ϕ_0.

F-3: Mutual admittance Y_{12} between two axial slots as a function of ϕ_0.

TABLE F-1

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 0$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
10^o	-63.59 -12^o	-67.11 -66^o	-72.13 178^o	-80.11 167^o
20^o	-67.13 -69^o	-72.57 173^o	-78.93 -75^o	-88.04 -110^o
30^o	-70.46 -131^o	-76.90 44^o	-83.98 26^o	-94.11 -32^o
45^o	-74.93 130^o	-82.36 -154^o	-90.41 -6^o	-102.17 82^o
60^o	-78.97 28^o	-87.25 6^o	-96.29 -39^o	-109.73 -164^o

TABLE F-2

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 0.5\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
0^o	-70.14 db 25^o	-70.11 25^o	-70.10 25^o	-70.09 26^o
10^o	-74.24 -20^o	-76.61 -94^o	-77.20 139^o	-81.28 144^o
20^o	-76.84 -101^o	-77.58 133^o	-80.64 -102^o	-88.34 -123^o
30^o	-77.48 -112^o	-79.63 10^o	-84.79 6^o	-94.24 -41^o
45^o	-79.13 90^o	-83.71 -179^o	-90.78 -19^o	-102.22 77^o
60^o	-81.68 -6^o	-88.04 -14^o	-96.49 -50^o	-109.76 -168^o

TABLE F-3

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 1\lambda$

ϕ_o	R = 1λ	R = 2λ	R = 4λ	R = 10λ
0^o	-86.65 db -173^o	-86.63 -172^o	-86.61 -172^o	-86.60 -172^o
10^o	-87.35 172^o	-87.92 134^o	-86.18 33^o	-84.30 78^o
20^o	-88.37 128^o	-86.51 26^o	-84.78 -180^o	-89.22 -160^o
30^o	-88.07 70^o	-85.58 -81^o	-86.95 -51^o	-94.63 -66^o
45^o	-87.12 -15^o	-87.09 109^o	-91.80 -60^o	-102.39 60^o
60^o	-87.51 -99^o	-90.13 -72^o	-97.06 -81^o	-109.84 179^o

TABLE F-4

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 2\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
0^o	-99.37 db -177^o	-99.34 -176^o	-99.33 -176^o	-99.33 -176^o
10^o	-99.72 176^o	-100.00 157^o	-98.96 93^o	-92.20 -144^o
20^o	-100.48 152^o	-99.39 85^o	-94.33 -67^o	-92.24 62^o
30^o	-100.83 115^o	-97.05 4^o	-93.13 109^o	-96.09 -163^o
45^o	-99.82 49^o	-95.40 -126^o	-95.21 150^o	-103.04 -7^o
60^o	-98.70 -15^o	-96.10 89^o	-99.12 161^o	-110.19 129^o

TABLE F-5

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 4\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
0^o	-111.56 db -178^o	-111.54 -178^o	-111.53 -178^o	-111.52 -178^o
10^o	-111.78 177^o	-111.97 168^o	-111.81 132^o	-105.41 -27^o
20^o	-112.38 164^o	-112.36 126^o	-108.23 21^o	-100.03 -35^o
30^o	-113.06 143^o	-111.16 67^o	-104.80 -104^o	-100.65 -154^o
45^o	-113.38 97^o	-108.48 -24^o	-103.49 28^o	-105.31 100^o
60^o	-112.64 47^o	-107.29 -123^o	-104.94 114^o	-111.46 -67^o

TABLE F-6

UI ASYMPTOTIC SOLUTIONS OF Y_{12} FOR $z_o = 8\lambda$

ϕ_o	$R = 1\lambda$	$R = 2\lambda$	$R = 4\lambda$	$R = 10\lambda$
0^o	-123.63 db -179^o	-123.61 -179^o	-123.61 -179^o	-123.60 -179^o
10^o	-123.78 -178^o	-123.92 173^o	-124.05 155^o	-120.63 54^o
20^o	-124.23 171^o	-124.56 150^o	-122.90 83^o	-113.13 -178^o
30^o	-124.87 159^o	-124.73 113^o	-119.69 -2^o	-110.52 -137^o
45^o	-125.87 131^o	-123.26 46^o	-116.53 -145^o	-111.57 -36^o
60^o	-126.32 94^o	-121.57 -21^o	-115.88 38^o	-115.46 -50^o

TABLE F-7

COMPARISON OF UI ASYMPTOTIC AND UI MODAL SOLUTIONS

z_o	ϕ_o	R = 1λ		R = 2λ	
		Modal	Asym.	Modal	Asym.
1λ	0^o	-87.06 db -171^o	-86.65 -173^o	-86.83 -172^o	-86.63 -172^o
	10^o	-87.69 176^o	-87.35 172^o	-88.23 139^o	-87.92 134^o
	20^o	-88.91 139^o	-88.37 128^o	-87.64 35^o	-86.51 26^o
	30^o	-89.40 85^o	-88.07 70^o	-87.01 -72^o	-85.77 -81^o
	45^o	-89.19 2^o	-87.32 -15^o	-88.67 119^o	-87.30 109^o
	60^o	-89.84 -83^o	-87.72 -99^o	-91.86 -61^o	-90.36 -72^o

TABLE F-8

COMPARISON OF UI ASYMPTOTIC AND UI MODAL SOLUTIONS

ϕ_o	z_o/λ	R = 1λ		R = 2λ		Planar
		Modal	Asym.	Modal	Asym.	(Exact)
0^o	0.5λ	--	-70.14 db 25^o	--	-70.11 25^o	-70.08 26^o
	1λ	-87.06 -171^o	-86.65 -173^o	-86.83 -172^o	-86.63 -172^o	-86.6 -172^o
	2λ	-99.97 -174^o	-99.37 -177^o	-99.61 -176^o	-99.34 -176^o	-99.32 -176^o
	4λ	-112.43 -175^o	-111.56 -178^o	-111.93 -177^o	-111.54 -178^o	-111.52 -178^o
	8λ	-124.33 -174^o	-123.63 -179^o	-124.12 -177^o	-123.61 -179^o	-123.60 -179^o

TABLE F-9

COMPARISON OF ASYMPTOTIC SOLUTIONS

z_0	ϕ_0	R = 2λ			R = 10λ		
		UI Asym.	PINY	OSU	UI Asym.	PINY	OSU
2λ	0°	-99.34 db -176°	-99.42 -172°	-105.44 -172°	-99.33 -176°	-99.41 -172°	-105.42 -172°
	10°	-100.00 157°	-99.93 164°	-105.37 152°	-92.2 -144°	-92.51 -142°	-92.53 -143°
	20°	-99.39 85°	-99.71 98°	-101.89 78°	-92.24 62°	-92.46 64°	-92.45 65°
	30°	-97.05 4°	-97.85 17°	-98.23 4°	-96.09 -163°	-96.30 -161°	-96.30 -160°
	45°	-95.40 -126°	-96.16 -115°	-96.09 -120°	-103.04 -7°	-103.25 -5°	-103.25 -4°
	60°	-96.10 89°	-96.68 97°	-96.6 96°	-110.19 129°	-110.41 131°	-110.41 131°

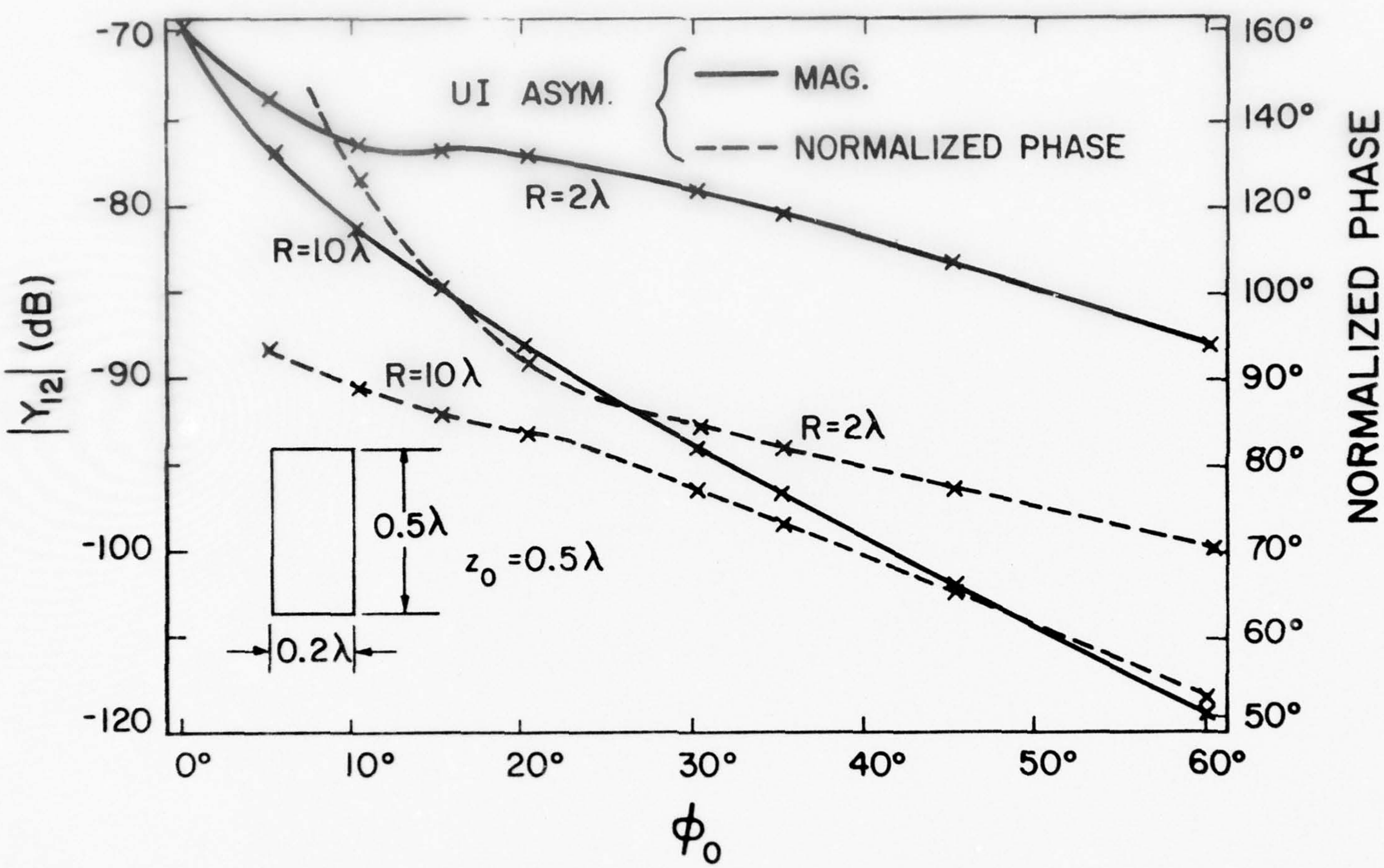

Figure F-1: Mutual admittance Y_{12} between two axial slots as a function of ϕ_0.

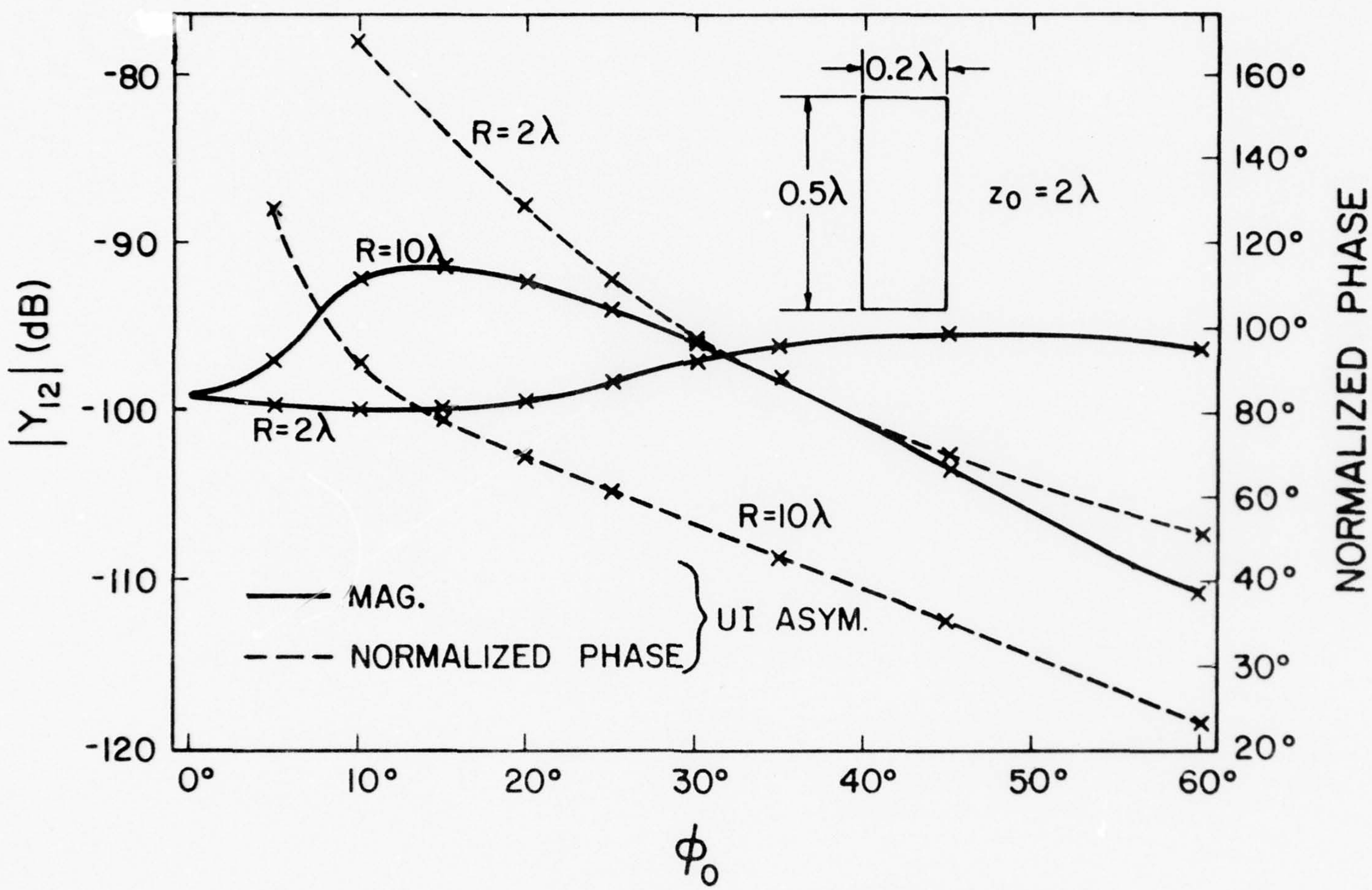

Figure F-2: Mutual admittance Y_{12} between two axial slots as a function of ϕ_0.

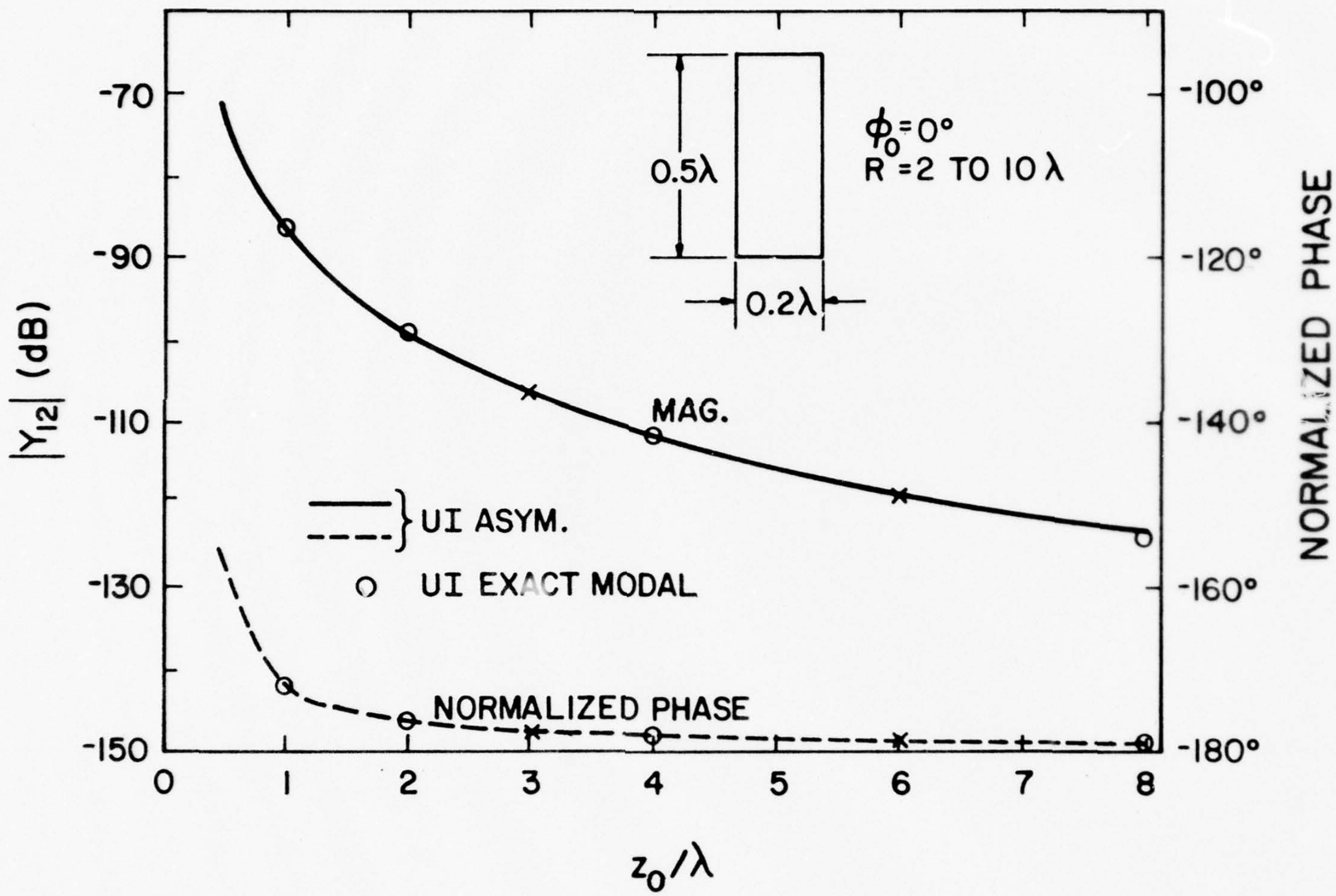

Figure F-3: Mutual admittance Y_{12} between two axial slots as a function of z_0.

APPENDIX B: COMPUTER PROGRAM LISTING

This appendix contains the program listing of all solutions, except the exact Hughes modal solution, discussed in the text.

ASYMPTOTIC SOLUTIONS OF Y_{12}

```
C THIS PROGRAM IS USED TO SOLVE FOR MUTUAL ADMITTANCE OF DOTS EITHER ON
C A CYLINDER OR ON A PLANE
C THERE ARE TWO TYPES OF SLOTS: 1) CIRCUMFERENTIAL; 2) AXIAL
C THIS PROGRAM INVOLVES A LOT OF INTERGRATIONS WHICH ARE BASICALLY SOLVED
C BY SUMMATION METHOD
C
C
C ***********************************
C *                                 *
C * INPUT PARAMETERS OF THIS PROGRAM *
C *                                 *
C ***********************************
C
C IPLAN-CONTROL THE PROGRAM IN DEALING WITH 2 DIFFERENT CASES:
C        1) IN PLANAR CASE, SET IPLAN=1
C        2) IN CYLINDRICAL CASE, SET IPLAN=2
C
C FOLLOWING PARAMETERS ARE COMMON TO BOTH IPLAN=1 AND IPLAN=2
C
C CUM  -ASSIGN A LOGIC VALUE 'TRUE' IF WE ARE INTERESTED IN THE
C       CIRCUMFERENTIAL CASE; OTHERWISE ASSIGN 'FALSE' TO IT
C AXIAL-ASSIGN A LOGIC VALUE 'TRUE' IF WE ARE INTERESTED IN THE AXIAL CASE;
C       OTHERWISE,ASSIGN 'FALSE' TO IT
C   A  -THE LONGER LENGTH OF A SLOT (MEASURED IN WAVELENGTH)
C   B  -THE SHORTER LENGTH OF A SLOT (MEASURED IN WAVELENGTH)
C IPA  -NUMBER OF SUBDIVISIONS OVER THE LONGER LENGTH
C IPB  -NUMBER OF SUBDIVISIONS OVER THE SHORTER LENGTH
C  UI  -UI=1 IF WE USE UI ASYMPTOTIC. IF NOT, UI=0
C OSU  -OSU=2 IF WE USE OSU ASYMPTOTIC. IF NOT, OSU=0
C PINY -PINY=3 IF WE USE PINY ASYMPTOTIC. IF NOT, PINY=0
C ZY1  -SELF ADMITTANCE OF SLOT(MEASURED IN MHO)
C   Z  -ARRAY OF 20 ELEMENTS AT MOST. EACH ELEMENT STANDS FOR THE
C       SEPARATION BETWEEN 2 POINTS ALONG Z AXIS (MEASURED IN WAVELENGTH)
C NDZ  -NUMBER OF ELEMENTS IN Z. DON'T BE GREATER THAN 20
C
C FOLLOWING PARAMETERS ARE ONLY GOOD FOR CYCLINDRICAL CASE.
C JUST FORGET THEM IF WE ARE DEALING WITH THE PLANAR CASE
C
C RADIUS - ARRAY OF DIFFERENT RADII OF A CYLINDER (MEASURED IN WAVELENGTH)
C  NDR   - NUMBER OF ELEMENTS IN RADIUS. DON'T BE GREATER THAN 6
C   PHI  - ARRAY OF 6 ELEMENTS AT MOST. EACH ELEMENT REPRESENTS THE ANGULAR
C          SEPARATION BETWEEN TWO SLOTS (MEASURED IN DEGREE)
C NDPHI  - NUMBER OF ELEMENTS IN PHI. DON'T BE GREATER THAN 6
C
C FOLLOWING PARAMETERS ARE FOR THE PLANAR CASE
C
C  YP  -ARRAY OF 20 ELEMENTS AT MOST. EACH OF THEM IS THE DISTANCE
C       BETWEEN 2 SLOTS ALONG Y-AXIS (MEASURED IN WAVELENGTH)
C  NDY -NUMBER OF ELEMENTS IN YP. DON'T BE GREATER THAN 20
C
C
      IMPLICIT COMPLEX*16 (C,H,Z)
      IMPLICIT REAL*8 (A-B,D-G,P-Y)
      REAL PHI(6),RADIUS(6),Z(20),FREQ(6),YP(20)
      INTEGER PINY,OSU,UI,TEST
      LOGICAL CUM,AXIAL
      REAL*8 KA,MAG,Z^
      REAL INCH
      REAL TN(10),TNPI(10)
      COMPLEX*16 DCMPLX
      COMMON/DATA1/TN,TNPI,RHO,C1,C2,F2,IOP
      COMMON PI,TZ1,TZ2,TY1,TY2,R,THETHA
      COMMON/DATA/AO,BO,ZO,YO
      READ(5,888) TN,TNPI
      WRITE(6,555)
C
C BEFORE EACH RUN,CHECK THE FOLLOWING PARAMETERS AND MAKE APPROPRIATE CORRECTION
C
C SET A VALUE TO THE NORMALIZATION FACTOR
      ZY1=1.
```

ASYMPTOTIC SOLUTIONS OF Y_{12}

```
C     THIS PROGRAM IS USED TO SOLVE FOR MUTUAL ADMITTANCE OF DOTS EITHER ON
C     A CYLINDER OR ON A PLANE
C     THERE ARE TWO TYPES OF SLOTS: 1)CIRCUMFERENTIAL; 2)AXIAL
C     THIS PROGRAM INVOLVES A LOT OF INTEGRATIONS WHICH ARE BASICALLY SOLVED
C     BY SUMMATION METHOD
C
C     ***********************************
C     *                                 *
C     *  INPUT PARAMETERS OF THIS PROGRAM *
C     *                                 *
C     ***********************************
C
C     IPLAN-CONTROL THE PROGRAM IN DEALING WITH 2 DIFFERENT CASES;
C           1) IN PLANAR CASE, SET IPLAN=1
C           2) IN CYLINDRICAL CASE, SET IPLAN=2
C
C     FOLLOWING PARAMETERS ARE COMMON TO BOTH IPLAN=1 AND IPLAN=2
C
C     CUM   -ASSIGN A LOGIC VALUE 'TRUE' IF WE ARE INTERESTED IN THE
C            CIRCUMFERENTIAL CASE; OTHERWISE ASSIGN 'FALSE' TO IT
C     AXIAL-ASSIGN A LOGIC VALUE 'TRUE' IF WE ARE INTERESTED IN THE AXIAL CASE;
C            OTHERWISE,ASSIGN 'FALSE' TO IT
C       A   -THE LONGER LENGTH OF A SLOT (MEASURED IN WAVELENGTH)
C       B   -THE SHORTER LENGTH OF A SLOT (MEASURED IN WAVELENGTH)
C     IPA   -NUMBER OF SUBDIVISIONS OVER THE LONGER LENGTH
C     IPB   -NUMBER OF SUBDIVISIONS OVER THE SHORTER LENGTH
C      UI   -UI=1 IF WE USE UI ASYMPTOTIC. IF NOT, UI=0
C     OSU   -OSU=2 IF WE USE OSU ASYMPTOTIC. IF NOT, OSU=0
C     PINY  -PINY=3 IF WE USE PINY ASYMPTOTIC. IF NOT, PINY=0
C     ZY1   -SELF ADMITTANCE OF SLOT(MEASURED IN MHO)
C       Z   -ARRAY OF 20 ELEMENTS AT MOST. EACH ELEMENT STANDS FOR THE
C            SEPARATION BETWEEN 2 POINTS ALONG Z AXIS (MEASURED IN WAVELENGTH)
C     NDZ   -NUMBER OF ELEMENTS IN Z. DON'T BE GREATER THAN 20
C
C     FOLLOWING PARAMETERS ARE ONLY GOOD FOR CYCLINDRICAL CASE.
C     JUST FORGET THEM IF WE ARE DEALING WITH THE PLANAR CASE
C
C     RADIUS - ARRAY OF DIFFERENT RADII OF A CYLINDER (MEASURED IN WAVELENGTH)
C       NDR  - NUMBER OF ELEMENTS IN RADIUS. DON'T BE GREATER THAN 6
C       PHI  - ARRAY OF 6 ELEMENTS AT MOST. EACH ELEMENT REPRESENTS THE ANGULAR
C              SEPARATION BETWEEN TWO SLOTS (MEASURED IN DEGREE)
C     NDPHI  - NUMBER OF ELEMENTS IN PHI. DON'T BE GREATER THAN 6
C
C     FOLLOWING PARAMETERS ARE FOR THE PLANAR CASE
C
C      YP   -ARRAY OF 20 ELEMENTS AT MOST. EACH OF THEM IS THE DISTANCE
C            BETWEEN 2 SLOTS ALONG Y-AXIS (MEASURED IN WAVELENGTH)
C     NDY   -NUMBER OF ELEMENTS IN YP. DON'T BE GREATER THAN 20
C
      IMPLICIT COMPLEX*16 (C,H,Z)
      IMPLICIT REAL*8 (A-B,D-G,P-Y)
      REAL PHI(6),RADIUS(6),Z(20),FREQ(6),YP(20)
      INTEGER PINY,OSU,UI,TEST
      LOGICAL CUM,AXIAL
      REAL*8 KA,MA,Z0
      REAL INCH
      REAL TN(10),TNPI(10)
      COMPLEX*16 DCMPLX
      COMMON/DATA1/TN,TNPI,RHO,C1,C2,T2,TOP
      COMMON ZT,TZ1,TZ2,TY1,TY2,P,THETHA
      COMMON/DATA/A0,B0,X0,Y0
      READ(5,888) TN,TNPI
      WRITE(6,555)
C
C     BEFORE EACH RUN,CHECK THE FOLLOWING PARAMETERS AND MAKE APPROPRIATE CORRECTION
C
C     SET A VALUE TO THE NORMALIZATION FACTOR
      ZY1=1.
```

```
C ASSIGN THE APPROPRIATE LOGIC VALUES TO AXIAL AND CUM
      CUM=.FALSE.
      AXIAL=.TRUE.
C SET A=THE LENGTH OF LONGER SIDE OF SLOT AND B=THE LENGTH OF SHORTER ONE
      A=0.5
      B=0.2
C CHOOSE A PROPER INTERGRATION GRID
      IPA=14
      IPB=2
C CHOOSE WHICH ASYMPTOTIC METHOD TO BE USED
      UI=0
      OSU=2
      PINY=3
C ASSIGN THE NUMBER OF DIFFERENT VALUES OF SEPARTION BETWEEN 2 SLOTS
C ALONG Z-DIRECTION.
C THEN CONSTRUCT A CORRESPONDING ARRAY Z OF NDZ ELEMENTS
      NDZ=1
      Z(1)=0.5
C SET IPLAN=1 FOR PLANAR CASE AND IPLAN=2 FOR CYLINDRICAL CASE.
      IPLAN=2
C IF IPLAN=1, SKIP THIS SECTION TO THE NEXT ONE
C IF IPLAN=2, SET THE NUMBER OF DIFFERENT RADII OF CYLINDER TO NDR
C AND THEN CONSTRUCT A CORRESPONDING ARRAY RADIUS OF NDR ELEMENTS
C SET NUMBER OF DIFFERENT VALUES OF ANGULAR SEPAPATION BETWEEN 2 SLOTS
C THEN CONSTRUCT A ARRAY OF PHI OF NDPHI ELEMENTS (MAX. NO. OF ELEMENTS IS 6)
      NDR=4
      RADIUS(1)=1.
      RADIUS(2)=2.
      RADIUS(3)=4.
      RADIUS(4)=10.
      NDPHI=6
      PHI(1)=0.
      PHI(2)=10.
      PHI(3)=20.
      PHI(4)=30.
      PHI(5)=45.
      PHI(6)=60.
C IF IPLAN=2 YOU CAN PUT THE WHOLE DECK INTO THE READER NOW
C IF IPLAN=1, SET THE NUMBER OF DIFFERENT VALUES OF DISTANCE BETWEEN 2 SLOTS
C ALONG Y-DIRECTION
C THE, CONSTRUCT A CORRESPONDING ARRAY YP OF NDY ELEMENTS
      NDY=1
      YP(1)=0.0001
C
C AFTER MAKING THE CORRECTIONS, YOU CAN PUT THE DECK INTO THE READER
C
      ICUM=1
      IAXIAL =2
      IF(.NOT.CUM) ICUM=2
      IF(.NOT.AXIAL) IAXIAL=1
      IL=1
      TEST=PINY-OSU-UI
      IF(TEST.EQ.3) IL=3
      IF(TEST.EQ.1.OR.TEST.EQ.-2) IL=2
      IU=MAXO(PINY,OSU,UI)
      PI=4*DATAN(1.DO)
      GO TO (1,2),IPLAN
    1 WRITE(6,777)
      GO TO 3
    2 WRITE(6,666)
    3 WRITE(6,555)
      XK=2.*PI
      DO 31 IU=ICUM,IAXIAL
      GO TO (4,5),IU
    4 AO=A*XK
      BO=B*XK
      GO TO 6
    5 SAVE=A
      A=B
      B=SAVE
```

```
C ASSIGN THE APPROPRIATE LOGIC VALUES TO AXIAL AND CUM
      CUM=.FALSE.
      AXIAL=.TRUE.
C SET A=THE LENGTH OF LONGER SIDE OF SLOT AND B=THE LENGTH OF SHORTER ONE
      A=0.5
      B=0.2
C CHOOSE A PROPER INTERGRATION GRID
      IPA=14
      IPB=2
C CHOOSE WHICH ASYMPTOTIC METHOD TO BE USED
      UT=0
      OSU=2
      PINY=3
C ASSIGN THE NUMBER OF DIFFERENT VALUES OF SEPARTION BETWEEN 2 SLOTS
C ALONG Z-DIRECTION.
C THEN CONSTRUCT A CORRESPONDING ARRAY Z OF NDZ ELEMENTS
      NDZ=1
      Z(1)=0.5
C SET IPLAN=1 FOR PLANAR CASE AND IPLAN=2 FOR CYLINDRICAL CASE.
      IPLAN=2
C IF IPLAN=1, SKIP THIS SECTION TO THE NEXT ONE
C IF IPLAN=2, SET THE NUMBER OF DIFFERENT RADII OF CYLINDER TO NDR
C AND THEN CONSTRUCT A CORRESPONDING ARRAY RADIUS OF NDR ELEMENTS
C SET NUMBER OF DIFFERENT VALUES OF ANGULAR SEPARATION BETWEEN 2 SLOTS
C THEN CONSTRUCT A ARRAY OF PHI OF NDPHI ELEMENTS (MAX. NO. OF ELEMENTS IS 6)
      NDR=4
      RADIUS(1)=1.
      RADIUS(2)=2.
      RADIUS(3)=4.
      RADIUS(4)=10.
      NDPHI=6
      PHI(1)=0.
      PHI(2)=10.
      PHI(3)=20.
      PHI(4)=30.
      PHI(5)=45.
      PHI(6)=60.
C IF IPLAN=2 YOU CAN PUT THE WHOLE DECK INTO THE READER NOW
C IF IPLAN=1, SET THE NUMBER OF DIFFERENT VALUES OF DISTANCE BETWEEN 2 SLOTS
C ALONG Y-DIRECTION
C THEN CONSTRUCT A CORRESPONDING ARRAY YP OF NDY ELEMENTS
      NDY=1
      YP(1)=0.0001
C
C AFTER MAKING THE CORRECTIONS, YOU CAN PUT THE DECK INTO THE READER
C
      ICUM=1
      IAXIAL =2
      IF(.NOT.CUM) ICUM=2
      IF(.NOT.AXIAL) IAXIAL=1
      IL=1
      TEST=PINY-OSU-UT
      IF(TEST.EQ.3) IL=3
      IF(TEST.EQ.1.OR.TEST.EQ.-2) IL=2
      IU=MAX0(PINY,OSU,UT)
      PI=4*DATAN(1.D0)
      GO TO (1,2),IPLAN
 1    WRITE(6,777)
      GO TO 3
 2    WRITE(6,666)
 3    WRITE(6,555)
      XK=2.*PI
      DO 30 IJ=ICUM,IAXIAL
      GO TO (4,5),IJ
 4    A0=A*XK
      B0=B*XK
      GO TO 6
 5    SAVE=A
      A=B
      B=SAVE
```

```
      AO=A*KK
      BO=B*KK
      ITEMP=IPB
      IPB=IPA
      IPA=TEEMP
6     WIDTH1=AO/IPA
      WIDTH2=30/IP3
      C1=CDEXP(DCMPLX(0.D0,-PI/3.))
      C2=CDEXP(DCMPLX(0.D0,PI/4.))
      F2=DSQRT(PI)
      Y1=-AO/2.-WIDTH1/2.
      Z1=-BO/2.-WIDTH2/2.
      GO TO (7,3),IPLAN
8     DO 40 IOP=IL,IU
      DO 50 IRAD=1,NDR
      RHO=XK*RADIUS(IRAD)
7     WRITE(6,444) XK
      IF(IJ.EQ.2) GO TO 9
      WRITE(6,333)
      GO TO 11
9     WRITE(6,222)
11    WRITE(6,111) A,AO,B,BO
      IF(IPLAN.EQ.1) GO TO 12
      WRITE(6,99) RADIUS(IRAD),RHO
      GO TO(91,92,93),IOP
91    WRITE(6,1233)
      GO TO 12
92    WRITE(6,1234)
      GO TO 12
93    WRITE(6,1235)
12    WRITE(6,66) IPA,IPB
      DO 60 JZ=1,NDZ
      ZO=Z(JZ)*KK
      IF(IPLAN.EQ.1) NDPHI=NDY
      DO 70 IY=1,NDPHI
      IF(IPLAN.EQ.1) GO TO 13
      YO=RHO*PI*PHI(IY)/180.
      IF(PHI(IY).EQ.0.0) YO=0.001
      Y=YO/KK
      GO TO 14
13    Y=YP(IY)
      YO=Y*XK
14    WRITE(6,77) PHI(IY),Y,Z(JZ),ZO
      Y2=YO-AO/2.-WIDTH1/2.
      Z2=ZO-BO/2.-WIDTH2/2.
      ZSUM=0.
      DO 60 K=1,IPA
      TY1=Y1+WIDTH1*K
      DO 90 L=1,IP3
      TZ1=Z1+WIDTH2*L
      DO 100 M=1,IPA
      TY2=Y2+WIDTH1*M
      DO 110 N=1,IPB
      TZ2=Z2+WIDTH2*N
      R=DSQRT((TY2-TY1)**2+(TZ2-TZ1)**2)
      THETHA=DATAN2((TZ2-TZ1),(TY2-TY1))
      IF(IPLAN.EQ.1) GO TO 60
      CALL CYLIND(IJ,ZSUM)
      GO TO 110
600   CALL PLANAR(IJ,ZSUM)
110   CONTINUE
100   CONTINUE
90    CONTINUE
80    CONTINUE
      ZY2=ZSUM*(WIDTH1*WIDTH2)**2*(-2.)/(AO*BO)
      MAG=CDABS(ZY2)
      PHASE=DATAN2(DIMAG(ZY2),DREAL(ZY2))*180./PI
      ZEXPON=CDEXP(DCMPLX(0.D0,DSQRT(YO**2+ZO**2)))
      ZPROD=ZY2*ZEXPON
      PHN=DATAN2(DIMAGZ(ZPROD),DREALZ(ZPROD))*180./PI
```

```
      DB=20.*DLOG10(CDABS(ZY2/ZY1))
      WRITE(6,78) MAG,PHASE,DB,PHN
70    CONTINUE
60    CONTINUE
      IF(IPLAN.EQ.1) GO TO 30
50    CONTINUE
49    CONTINUE
30    CONTINUE
66    FORMAT(' ',3X,'*  NORMALIZATION: ABS(Y11)=1 (UNLESS SPECIFIED OTHE
     1RWISE)'/' ',3X,'*  INTEGRATION GRID: ',I3,' X',I3/
     1         '0',10X,'***** DATA OUTPUT *****')
77    FORMAT('0','PHI= ',F6.3,' <DEG>:',5X,'Y= ',D15.7,' <WAVELENGTH>:'
     1       5X,'Z= ',E10.4,' <WAVELENGTH>:',5X,'KZ= ',E10.4)
78    FORMAT(1X,'Y12= ',E13.4,' <MHO>',F7.2,' <DEG>:',5X,'DB= ',E12.5,
     1       ':        NORM PHASE= ',F7.2,' <DEG>')
99    PORMAT(' ',3X,'*  CYLINDER: ',5X,'P= ',E10.4,' <WAVELENGTH>',
     1       5X,'KR= ',E10.4/' ',3X,'* METHOD OF SOLUTION:')
111   FORMAT(' ',3X,'*  SLOT DIMENSION: '/
     1       ' ',6X,'A=',E10.4,' <WAVELENGTH>',5X,'KA= ',E10.4/
     1       ' ',6X,'B=',E10.4,' <WAVELENGTH>',5X,'KB= ',E10.4)
222   FORMAT(' ',3X,'*  AXIAL')
333   FORMAT(' ',3X,'*  CIRCUMFERENTIAL')
444   FORMAT('-',3X,'*  K= ',E10.4,' <1/WAVELENGTH>')
555   FORMAT('0',10X,'***************************************************')
666   FORMAT('0',10X,'*  MUTUAL ADMITTANCE OF A SLOT ON A CYLINDER  *')
777   FORMAT('0',10X,'*   MUTUAL ADMITTANCE OF A SLOT ON A PLANE    *')
888   FORMAT(10F8.5/10F8.5)
1233  FORMAT('+',26X,' UI ASYMPTOTIC')
1234  FORMAT('+',26X,' OSU ASYMPTOTIC')
1235  FORMAT('+',26X,'PINY ASYMPTOTIC')
      STOP
      END
C
C THIS SUBROUTINE IS USED TO CALCULATE THE FUNCTIONS CVF,CUF,CV1F,CVPF,CUPF
C
      SUBROUTINE FOCK(X)
      IMPLICIT COMPLEX*16 (C,Z)
      IMPLICIT REAL*8 (A-B,D-H,P-Y)
      REAL TN(10),TNPI(10)
      COMPLEX*16 DCMPLX
      COMMON/CF/CVF,CUF,CV1F,CVPF,CUPF
      COMMON PI,TZ1,TZ2,TY1,TY2,R,THETHA
      COMMON/DATA1/TN,TNPI,RHO,C1,C2,F2
      F1=DSQRT(X)
      F3=X**(3./2.)
      CVF=0.
      CUF=0.
      CV1F=0.
      CVPF=0.
      CUPF=0.
      DO 20 N=1,10
      ZTN=TN(N)*C1
      ZTNPI=TNPI(N)*C1
      C3=CDEXP(DCMPLX(0.D0,-X)*ZTNPI)
      C4=CDEXP(DCMPLX(0.D0,-X)*ZTN)
      CVF=CVF+C3/ZTNPI
      CUF=CUF+C4
      CV1F=CV1F+C3
      CVPF=(1.D0-DCMPLX(0.D0,2*X)*ZTNPI)*C3/ZTNPI+CVPF
      CUPF=(1.D0-DCMPLX(0.D0,2*X/3.)*ZTN)*C4+CUPF
20    CONTINUE
      CVF=F2*F1*CVF/C2
      CUF=2.*F2*F3*C2*CUF
      CV1F=2.*F2*F3*C2*CV1F
      CVPF=F2*CVPF/(2.*F1*C2)
      CUPF=3.*F2*F1*C2*CUPF
      RETURN
      ENTRY FOCK1(X)
      F1=DSQRT(X)
      F3=X**3
      Z1=F2*C2*DSQRT(F3)
```

```
      Z2=DCMPLX(0.D0,1.D0/60.)*X**3
      Z3=F2*X**(9./2.)/(C2*64.)
      F4=E3**2
      CVF=1.D0-Z1/4.+7*Z2+7.*Z3/8.-4.141D-3*F4
      CUF=1.D0-Z1/2.+25.*Z2+5.*Z3-3.701D-2*F4
      CV1F=1.D0+Z1/2.-35.*Z2-7.*Z3+4.555D-2*F4
      CVPF=3.*F2*Z1/(8.*C2**3)+21.*Z2/X+63*Z3/(16.*X)-2.485D-2*F4/X
      CUPF=3.*F2*F1/(4.*C2**3)+DCMPLX(0.D0,5.D0/4.D0)*X**2+45.*Z3/(2*X)
     1  -2.221D-1*F4/X
      RETURN
      END
C
C THIS SUBROUTINE IS USED TO GET THE 'PLANAR' SOLUTION
C
      SUBROUTINE PLANAR(IJ,ZSUM)
      IMPLICIT COMPLEX*16 (C,H,Z)
      IMPLICIT REAL*8 (A-B,D-G,P-Y)
      REAL*8 Z)
      COMMON PI,TZ1,TZ2,TY1,TY2,R,THETHA
      COMMON/DATA/A,B,Z0,Y0
      GO TO (10,20),IJ
   10 XM1=((TZ2-TZ1)/R)**2
      XM2=2.-3.*XM1
      XRL=XM2/3
      XIM=XM1+XM2/R**2
      HA=CDEXP(DCMPLX(0.D0,-R))*DCMPLX(XRL,-XIM)/(240.*R*PI**2)
      FACTOR=DCOS(PI*TY1/A)*DCOS(PI*(TY2-Y0)/A)
      ZSUM=ZSUM+FACTOR*HA
      RETURN

   20 ZA1=DCMPLX(1./R**2,1./R)*(2.-3.*DCOS(THETEA)**2)
      ZA2=CDEXP(DCMPLX(0.D0,-R))*(DCOS(THETHA)**2+ZA1)/R
      HA=(0.,-1.)*ZA2/(240.*PI**2)
      FACTOR=DCOS(PI*TZ1/B)*DCOS(PI*(TZ2-Z0)/B)
      ZSUM=ZSUM+FACTOR*HA
      RETURN
      END
C
C THIS SUBROUTINE IS USED TO GET THE 'CYLINDEICAL' SOLUTION
C
      SUBROUTINE CYLIND(IJ,ZSUM)
      IMPLICIT COMPLEX*16 (C,H,Z)
      IMPLICIT REAL*8 (A-B,D-G,P-Y)
      REAL*8 Z0,KA
      REAL TN(10),TNPI(10)
      COMMON PI,TZ1,TZ2,TY1,TY2,R,THETHA
      COMMON/CF/CVF,CUF,CV1F,CVPF,CUPF
      COMMON/DATA1/TN,TNPI,RHO,C1,C2,F2,IOP
      COMMON/DATA/A,B,Z0,Y0
      ZGR=(0.,-1.)*CDEXP(DCMPLX(0.D0,-R))/(240.*R*PI**2)
      ANGLE=DATAN2(DABS(TZ2-TZ1),DABS(TY2-TY1))*180./PI
      IF(ANGLE.LT.89.99) GO TO 10
      IT=1
      THETHA=PI*89.99/180.
      ZW=(0.,1.)/R+CDEXP(DCMPLX(0.D0,-PI/4.D0))*DSQRT(PI*R/2.)/RHO
   10 RHOG=RHO/DCOS(THETHA)**2
      KA=R*DABS((1./(2.*RHOG**2))**(1./3.))
      IF(KA.LT.0.7) GO TO 20
      CALL FOCK(KA)
      GO TO 30
   20 CALL FOCK1(KA)
   30 IF(IT.EQ.1) GO TO 40
      ZN=(0.,1.)*(1./DCOS(THETHA)**2)/R*(CUF-CV1F*DSIN(THETHA)**2)
   40 GO TO (21,22,23),IOP
   21 ZHB1=DCMPLX(1.D0,-1./R)*CVF
      ZHB2=CUF/R**2
      ZHB3=(0.,1.)*CVPF/(DSQRT(2.D0)*RHOG)**(2./3.)
      ZHB4=(0.,1.)*(1./(DSQRT(2.D0)*RHOG))**(2./3.)*CUPF*DTAN(THETHA)**2
      HB=ZGR*(ZHB1-ZHB2+ZHB3+ZHB4)
      ZTM1=(0.,1.)*CUPF/(DSQRT(2.D0)*RHOG)**(2./3.)
      HT=(0.,1.)*ZGR/R*(CVF+DCMPLX(1.D0,-2./R)*CUF+ZTM1)
```

```
      HZ=HB*DCOS(THETHA)**2+HT*DSIN(THETHA)**2
      HPHI=HB*DSIN(THETHA)**2+HT*DCOS(THETHA)**2
      GO TO 500
22    HB=ZGR*CVF
      HT=(0.,1.)*ZGR*CVF/R
      HZ=HB*DCOS(THETHA)**2+HT*DSIN(THETHA)**2
      HPHI=HB*DSIN(THETHA)**2+HT*DCOS(THETHA)**2
      GO TO 500
23    ZTM2=(0.,1.)*(1.-3.*DSIN(THETHA)**2)/R
      HZ=ZGR*CVF*(DCOS(THETHA)**2+(0.,1.)*(2.-3.*DCOS(THETHA)**2)/R)
      HPHI=ZGR*(CVF*(DSIN(THETHA)**2+ZTM2)+ZW)
500   ZGREEN=HPHI
      IF(IJ.EQ.2) ZGREEN=HZ
      FACTOR=DCOS(PI*TY1/A)*DCOS(PI*(TY2-Y0)/A)
      IF(IJ.EQ.2) FACTOR=DCOS(PI*TZ1/B)*DCOS(PI*(TZ2-Z0)/B)
      ZSUM=ZSUM+FACTOR*ZGREEN
      RETURN
      END
```

UI EXACT MODAL SOLUTION OF Y_{12}

```
C     PROGRAM FOR COMPUTATION OF THE MUTUAL ADMITTANCE BETWEEN TWO
C     IDENTICAL CIRCUMFERENTIAL SLOTS ON A CYLINDER(UI MODAL SOLUTION)
      REAL   K0,KZ,KT,I2,KZKIRO
      COMPLEX*16  Z1,Y12,ESIEXP,YN12
      REAL*8    FI(400), FM(400),FN(400),DIMAG,DREAL,DATAN2
C     INPUT PARAMETERS :
C     K0=WAVE NUMBER IN FREE SPACE IN TERMS OF 1/INCH
C     RHO=RADIUS OF CYLINDER <INCH>
C     A*B= SLOT DIMENSION    , A>B     <INCH>
C     PHI0=ANGULAR SEPARATION OF THE SLOTS (CENTER TO CENTER) <RADIAN>
C     Z0= SEPARATION OF THE SLOTS IN Z-DIRECTION    <INCH>
C     Y11= NORMALIZATION FACTOR
C     EMAX= MAXIMUM NO. OF TERMS WHICH HAS BEEN USED IN CALCULATION OF
C     INFINITE SERIES
C     NCYCLE=NO. OF SUBSECTIONS BETWEEN ANY TWO  SUCCESSIVE ZEROS OF INTEGRAND
C     IN TRAPEZOIDAL RULE FOR NUMERICAL INTEGRATION
      PI=3.14159265
      Y0=1./(120.*PI)
      K0=2.*PI
      FREQ=3.E10*K0/(2.*PI*2.54)
      A=0.5
      B=0.2
      AKA=K0*A
      BKB=K0*B
      NMAX=380
      NCYCLE=40
      Y11=1.
      RHO=2.
      RK=K0*RHO
      WRITE(6,30)
   30 FORMAT(40X,'MUTUAL ADMITTANCE OF SLOTS ON A CYLINDER'/)
      WRITE(6,50)
   50 FORMAT(52X,'CIRCUMFERENTIAL'/)
      WRITE(6,51) FREQ,K0
   51 FORMAT(25X,'FREQUENCY',10X,'F=',E12.5,'<HZ>',10X,'K=',E12.5,
     ,'<1/INCH>'/)
      WRITE(6,52)
   52 FORMAT(10X,'SLOT DIMENSIONS')
      WRITE(6,53) A,AKA,B,BKB
   53 FORMAT(10X,'A=',E10.4,'<INCH>',4X,'KA=',E10.4,15X,'B=',E10.4,
     ,'<INCH>',4X,'KB=',E10.4/)
      WRITE(6,54) RHO,RK
   54 FORMAT(//35X,'CYLINDER',10X,'R=',E12.5,'<INCH>',4X,'KR=',E12.5/)
      WRITE(6,55)
   55 FORMAT(35X,'METHOD OF SOLUTION   *MODAL*'/)
      WRITE(6,56)  Y11
   56 FORMAT(50X,'NORMALIZATION |Y11|=',E10.4///)
      WRITE(6,57)
   57 FORMAT(55X,'DATA OUTPUT'/)
      PHI0=PI/6.
      Z0=2.
C     PHIB=HALF ANGULAR WIDTH OF THE SLOT
      PHIB=ARSIN(A/(2.*RHO))
```

```
C     COMPUTATION OF INFINITE SERIES
      MMAX12=MMAX+1
      DO 100 M=1,MMAX12
      M1=M-1
      EPM=1.
      IF(M.EQ.1) EPM=2.
      PHIB1=PHIB
      IF(ABS(PHIB*M1-PI/2.).LE.1.E-7) PHIB1=PHIB*1.001
  100 F1(M)=COS(M1*PHIO)*(-PI*COS(M1*PHIB1)/(((M1*PHIB1)**2-(PI/2.)**2
     &))))**2*(1./EPM)
C     INTEGRATION OF PSI(KZ)*R1(M,KZ)*EXP(-J*KZ*ZO) BETWEEN 0 AND KO
C     DELTA= NEIGHBOURHOOD OF THE SINGULAR POINT KZ=KO IN WHICH THE INTEGRAL
C     HAS BEEN CALCULATED ANALYTICALLY
      DELTA=0.0001*KO
C     DELTA1= NEIGHBOURHOOD OF THE SINGULAR POINT KZ=KO WHERE THE INTEGRAND
C     VARIES RAPIDLY AND 'NDELTA' SAMPLES HAVE BEEN USED.
      DELTA1=0.01*KO
      NDELTA=100
      DKZ2=(DELTA1-DELTA)/NDELTA
C     NSECT1= NO. OF SUBSECTIONS BETWEEN  0 AND KO-DELTA1
      NSECT1=(IFIX((B+ZO+RHO)*KO/PI)+2)*NCYCLE
      DKZ1=(KO-DELTA1)/NSECT1
      NSECT=NSECT1+NDELTA+2
C     I1=FIRST INTEGRAL (BETWEEN 0. AND KO)
      I1=(0.,0.)
      DO 200 I=1,NSECT
      IF(I.LE.NSECT1+1) GO TO 120
      KZ=KO-DELTA1+(I-NSECT1-2)*DKZ2
      DKZ=DKZ2
      GO TO 140
  120 KZ=(I-1)*DKZ1
      IF(KZ.EQ.0) KZ=0.00001*KO
      DKZ=DKZ1
  140 CIN=1.
      IF((I.EQ.1).OR.(I.EQ.NSECT1+1).OR.(I.EQ.NSECT1+2).OR.(I.EQ.NSECT)
     &) CIN=0.5
      KT=SQRT(KO*KO-KZ*KZ)
      PSIEXP=(SIN(KZ*B/2.)/(KZ*B/2.))**2*CIN*DKZ*CEXP((0.,-1.)*KZ*ZO)
      MMAX1=MMAX
      ROKT=RHO*KT
C     COMPUTATION OF FM(N)=1./(JN(X)**2+YN(X)**2) AND FN(N)=1./(DJN(X)**2+
C     DYN(X)**2) FOR X=ROKT AND N=0 TO MMAX1 ; WHERE MMAX1 IS A NUMBER AFTER
C     WHICH  THE CONTRIBUTIONS OF FM(N) AND FN(N) TO THE INFINITE SUM
C     BECOME NEGLIGIBLE. MMAX1 IS A FUNCTION OF  THE ARGUMENT X AND IS ALWAYS
C     LESS THAN OR EQUAL TO MMAX. MMAX1, FM(N) AND FN(N) ARE CALCULATED
C     BY SUBROUTINE FMFN(X,MMAX1,FM,FN).
      CALL FMFN(ROKT,MMAX1,FM,FN)
      KZKTRO=(KZ/(KT*KO*RHO))**2
      DO 200 M=1,MMAX1
      M1=M-1
      R1=(1./KT**2)*(FM(M)+M1**2*KZKTRO*FN(M))
  200 I1=I1+R1*PSIEXP*F1(M)
      I1=(2.*KO/(PI*RHO))*(I1-F1(1)*CEXP((0.,-1.)*KO*ZO)*(PI*PI/(2.*KO))
```

```
     3*(SIN(KO*B/2.)/(KO*B/2.))**2/(2.*(0.5772156649+ALOG(RHO*SQRT
     3(KO/2.)))+ALOG(DELTA)))
C     COMPUTATION OF I2 (BETWEEN ZERO AND ETAMAX ; WHERE ETAMAX  IS A NUMBER
C     AFTER WHICH THE INTEGRAND BECOMES VERY SMALL)
      I2=0.
      ETAMAX=14./(ZO-B)
C     THE INTEGRATION IS CARRIED OUT BY TRAPEZOIDAL RULE. AT FIRST THE WHOLE
C     RANGE OF INTEGRATION (0.,ETAMAX) IS DEVIDED INTO TWO SUBINTERVALS :
C     (0.,ETA1) AND (ETA1,ETAMAX) , WHERE ETA1=ETAMAX/2.. THEN THE NUMERICAL
C     COMPUTATION OF THE  INTEGRAL IS PERFORMED IN THESE SUBINTERVALS WITH THE
C     NO. OF SAMPLES IN THE FIRST SUBINTERVAL TWO TIMES THAT IN THE SECOND ONE.
      ETA1=7./(ZO-B)
      NSECT1=(IFIX(SQRT(KO*KO+ETA1**2)*RHO/PI)+2)*NCYCLE
      DETA1=ETA1/NSECT1
      DETA2=2.*DETA1
      NSECT2=IFIX((ETAMAX-ETA1)/DETA2)+1
      NSECT=NSECT1+NSECT2+2
      DO 300 I=1,NSECT
      IF(I.LE.NSECT1+1) GO TO 220
      ETA=ETA1+(I-NSECT1-2)*DETA2
      DETA=DETA2
      GO TO 240
  220 ETA=(I-1)*DETA1
      IF(ETA.EQ.0.) ETA=0.0001/B
      DETA=DETA1
  240 CIN=1.
      IF((I.EQ.1).OR.(I.EQ.NSECT1+1).OR.(I.EQ.NSECT1+2).OR.(I.EQ.NSECT)
     3) CIN=0.5
      PSEX=(SINH(ETA*B/2.)/(ETA*B/2.))**2*EXP(-ETA*ZO)*DETA*CIN
      KT=SQRT(KO*KO+ETA**2)
      ETKTRO=(ETA/(RHO*KO*KT))**2
      MMAX1=MMAX
      CALL FMFN(RHO*KT,MMAX1,FM,FN)
      DO 300 M=1,MMAX1
      M1=M-1
      R1=(1/(KT*KT))*(FM(M)-M1*M1*ETKTRO*FN(M))
  300 I2=I2+F1(M)*R1*PSEX
      I2=I2*2.*KO/(PI*RHO)
      Y12=(I1+(0.,1.)*I2)*A*B*YO/(2.*PI*PI*RHO)
C     NORMALIZATION OF THE PHASE OF Y12
      YN12=Y12*CEXP((0.,1.)*(KO*SQRT(ZO*ZO+(RHO*PHIO)**2)))
C     COMPUTATION OF THE ACTUAL PHASE 'PHASEY' AND NORMALIZED PHASE 'PHASNM'
C     OF Y12.
      PHASEY=DATAN2(DIMAG(Y12),DREAL(Y12))*180./PI
      PHASNM=DATAN2(DIMAG(YN12),DREAL(YN12))*180./PI
C     COMPUTATION OF THE MAGNITUDE OF THE Y12 IN TERMS OF <MHO> AND <DB>.
      AMPY=CDABS(Y12)
      AMPYDB=ALOG10(AMPY/ABS(Y11))*20.
      RPHIK=KO*RHO*PHIO
      ZOK=KO*ZO
      PHIOD=PHIO*180./PI
      WRITE(6,400) PHIOD,RPHIK,ZO,ZOK,AMPY,PHASEY,AMPYDB,PHASNM
  400 FORMAT(20X//10X,'PHIO=',F7.2,'<DEG>',4X,'K*R*PHIO=',E10.4
     3,16X,'ZO=',E10.4,'<INCH>',4X,'K*ZO=',E10.4/10X,'Y12=',E12.5,
```

```
     ,'<MHO>',3X,F7.2,'<DEG>',10X,'DB=',E12.5,3X,'NORM. PHASE=',F7.2)
      STOP
      END
C     PROGRAM TO COMPUTE THE MUTUAL ADMITTANCE BETWEEN TWO IDENTICAL
C     AXIAL SLOTS ON A CYLINDER ( UI MODAL SOLUTION)
      REAL   KO,KZ,KI,IZ,KZKTRO
      COMPLEX*16  I1,Y12,PSIEXP,YN12
      REAL*8    F1(400) , FM(400),FN(400),DIMAG,DREAL,DATAN2
C     INPUT PARAMETERS:
C     KO=WAVE NUMBER IN FREE SPACE IN TERMS OF 1/INCH
C     NCYCLE=NO. OF SUBSECTIONS BETWEEN ANY TWO  SUCCESSIVE ZEROS OF INTEGRAND
C     IN TRAPEZOIDAL RULE FOR NUMERICAL INTEGRATION
C     A*B= SLOT DIMENSION   B>A      <INCH>
C     RHO=RADIUS OF CYLINDER  <INCH>
C     PHIO=ANGULAR SEPARATION OF THE SLOTS (CENTER TO CENTER) <RADIAN>
C     ZO= SEPARATION OF THE SLOTS IN Z-DIRECTION    <INCH>
C     Y11= NORMALIZATION FACTOR
C     MMAX= MAXIMUM NO. OF TERMS WHICH HAS BEEN USED IN CALCULATION OF
C     INFINITE SERIES
      MMAX=380
      PI=3.14159265
      YO=1./(120.*PI)
      KO=2.*PI
      FREQ=3.E10*KO/(2.*PI*2.54)
      NCYCLE=100
      B=0.5
      A=0.2
      AKA=KO*A
      BKB=KO*B
      RHO=2.
      RK=KO*RHO
      Y11=1.
      WRITE(6,30)
   30 FORMAT(40X,'MUTUAL ADMITTANCE OF SLOTS ON A CYLINDER'/)
      WRITE(6,50)
   50 FORMAT(57X,'AXIAL'/)
      WRITE(6,51) FREQ,KO
   51 FORMAT(25X,'FREQUENCY',10X,'F=',E12.5,'<HZ>',10X,'K=',E12.5,
     ,'<1/INCH>'/)
      WRITE(6,52)
   52 FORMAT(10X,'SLOT DIMENSIONS')
      WRITE(6,53) A,AKA,B,BKB
   53 FORMAT(10X,'A=',E10.4,'<INCH>',4X,'KA=',E10.4,15X,'B=',E10.4,
     ,'<INCH>',4X,'KB=',E10.4/)
      WRITE(6,54) RHO,RK
   54 FORMAT(35X,'CYLINDER',10X,'R=',E12.5,'<INCH>',4X,'KR=',E12.5/)
      WRITE(6,55)
   55 FORMAT(35X,'METHOD OF SOLUTION   *MODAL*'/)
      WRITE(6,56)  Y11
   56 FORMAT(50X,'NORMALIZATION |Y11|=',E10.4///)
      WRITE(6,57)
   57 FORMAT(55X,'DATA OUTPUT'/)
      PHIO=PI/4.
      ZO=2.
```

```
C     PHIA=HALF ANGULAR WIDTH OF THE SLOT
      PHIA=2.*ARSIN(A/(2.*RHO))
C     COMPUTATION OF INFINITE SERIES
      MMAX12=MMAX+1
      DO 100 M=1,MMAX12
      M1=M-1
      IF(M.EQ.1) GO TO 99
      F1(M)=COS(M1*PHIO)*(SIN(M1*PHIA/2.)/(M1*PHIA/2.))**2
      GO TO 100
   99 F1(M)=0.5
  100 CONTINUE
C     INTEGRATION OF PSI(KZ)*F1(M,KZ)*EXP(-J*KZ*ZC) BETWEEN 0 AND KO
C     DELTA= NEIGHBOURHOOD OF THE SINGULAR POINT KZ=KO IN WHICH THE INTEGRAL
C     HAS BEEN CALCULATED ANALYTICALLY
      DELTA=1.E-7*KO
C     NSECT1=NO. OF SAMPLES IN THE INTERVAL (C.,KO-DELTA).
      NSECT1=(IFIX((B+ZC+RHO)*KO/PI)+2)*NCYCLE
      DKZ=(KO-DELTA)/NSECT1
      NSECT=NSECT1+1
      I1=(0.,0.)
C     I1=FIRST INTEGRAL (BETWEEN 0. AND KO)
      DO 200 I=1,NSECT
      KZ=(I-1)*DKZ
      IF(KZ.EQ.0) KZ=0.00001*KO
      CIN=1.
      IF((I.EQ.1).OR.(I.EQ.NSECT)) CIN=0.5
      KT=SQRT(KO*KO-KZ*KZ)
      IF(ABS(KZ*B/2.-PI/2.).LE.1.E-8) KZ=1.000001*KZ
      PSIEXP=(COS(KZ*B/2.)/((KZ*B/2.)**2-(PI/2.)**2))**2*CIN*DKZ
     $*CEXP((0.,-1.)*KZ*ZC)
      MMAX1=MMAX
      ROKT=RHO*KT
C     COMPUTATION OF FM(N)=1./(JN(X)**2+YN(X)**2) AND FN(N)=1./(DJN(X)**2+
C     DYN(X)**2) FOR X=ROKT AND N=0 TO MMAX1 ; WHERE MMAX1 IS A NUMBER AFTER
C     WHICH THE CONTRIBUTIONS OF FM(N) AND FN(N) TO THE INFINITE SUM
C     BECOME NEGLIGIBLE. MMAX1 IS A FUNCTION OF THE ARGUMENT X AND IS ALWAYS
C     LESS THAN OR EQUAL TO MMAX. MMAX1, FM(N) AND FN(N) ARE CALCULATED
C     BY SUBROUTINE FMFN(X,MMAX1,FM,FN).
      CALL FMFN(ROKT,MMAX1,FM,FN)
      DO 200 M=1,MMAX1
      M1=M-1
  200 I1=I1+FN(M)*PSIEXP*F1(M)
C     COMPUTATION OF I2 (BETWEEN ZERO AND ETAMAX ; WHERE ETAMAX IS A NUMBER
C     AFTER WHICH THE INTEGRAND BECOMES VERY SMALL)
      I2=0.
      ETAMAX=14./(ZO-B)
C     THE INTEGRATION IS CARRIED OUT BY TRAPEZOIDAL RULE. AT FIRST THE WHOLE
C     RANGE OF INTEGRATION (0.,ETAMAX) IS DEVIDED INTO TWO SUBINTERVALS :
C     (0.,ETA1) AND (ETA1,ETAMAX) , WHERE ETA1=ETAMAX/2.. THEN THE NUMERICAL
C     COMPUTATION OF THE INTEGRAL IS PERFORMED IN THESE SUBINTERVALS WITH THE
C     NO. OF SAMPLES IN THE FIRST SUBINTERVAL TWO TIMES THAT IN THE SECOND ONE.
      ETA1=7./(ZO-B)
      NSECT1=(IFIX(SQRT(KO*KO+ETA1**2)*RHO/PI)+2)*NCYCLE
      DETA1=ETA1/NSECT1
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```
      DETA2=2.*DETA1
      NSECT2=IFIX((ETAMAX-ETA1)/DETA2)+1
      NSECT=NSECT1+NSECT2+2
      DO 300 I=1,NSECT
      IF(I.LE.NSECT1+1) GO TO 220
      ETA=ETA1+(I-NSECT1-2)*DETA2
      DETA=DETA2
      GO TO 240
  220 ETA=(I-1)*DETA1
      IF(ETA.EQ.0.) ETA=0.0001/A
      DETA=DETA1
  240 CIN=1.
      IF((I.EQ.1).OR.(I.EQ.NSECT1+1).OR.(I.EQ.NSECT1+2).OR.(I.EQ.NSECT)
     $) CIN=0.5
      PSEX=(COSH(ETA*B/2.)/((ETA*B/2.)**2+(PI/2.)**2))**2*DETA*CIN
     $*EXP(-ETA*ZO)
      KT=SQRT(KO*KO+ETA**2)
      MMAX1=MMAX
      CALL FMFN(RHO*KT,MMAX1,FM,FN)
      DO 300 M=1,MMAX1
      M1=M-1
  300 I2=I2+FN(M)*PSEX*F1(M)
      Y12=(I1+(0.,1.)*I2)*A*B*YO/(PI*KC*RHO**2)
C     NORMALIZATION OF THE PHASE OF Y12
      YN12=Y12*CEXP((0.,1.)*(KO*SQRT(ZO*ZO+(RHO*PHIO)**2)))
C     COMPUTATION OF THE ACTUAL PHASE 'PHASEY' AND NORMALIZED PHASE 'PHASNM'
C     OF Y12.
      PHASEY=DATAN2(DIMAG(Y12),DREAL(Y12))*180./PI
      PHASNM=DATAN2(DIMAG(YN12),DREAL(YN12))*180./PI
C     COMPUTATION OF THE MAGNITUDE OF THE Y12 IN TERMS OF <MHO> AND <DB>.
      AMPY=CDABS(Y12)
      AMPYDB=ALOG10(AMPY/ABS(Y11))*20.
      RPHIK=KO*RHO*PHIO
      ZOK=KO*ZO
      PHIOD=PHIO*180./PI
      WRITE(6,400) PHIOD,RPHIK,ZO,ZOK,AMPY,PHASEY,AMPYDB,PHASNM
  400 FORMAT(20X//10X,'PHIO=',F7.2,'<DEG>',4X,'K*R*PHIO=',E10.4
     $,16X,'ZO=',E10.4,'<INCH>',4X,'K*ZO=',E10.4/10X,'Y12=',E12.5,
     $'<MHO>',3X,F7.2,'<DEG>',10X,'DB=',E12.5,3X,'NORM. PHASE=',F7.2)
      STOP
      END
      SUBROUTINE FMFN(X,N,FM,FN)
      REAL*8 DUM1(400),DUM2(400)
      REAL*8 FJ(400),XB,BSSY(400),FM(400),FN(400)
      PI=3.14159265
      XB=X
      IF(X-0.1) 10,10,20
   10 GAMLOG=ALOG(X/2.)+0.5772156649
      X2=X*X
      X3=X2*X
      X4=X*X3
      X5=X*X4
      BSSY1=2.*(GAMLOG*(1.-X2/4.+X4/64.)+X2/4.-3.*X4/128.)/PI
      BSSY2=-2./(PI*X)+2.*(GAMLOG*(X/2.-X3/16.+X5/384.)-X/4.+1.25*X3/16.
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      ,-3.33333*X5/768.)/PI
      GO TO 25
   20 CALL BESY(X,0,BSSY1,IER)
      CALL BESY(X,1,BSSY2,IER)
   25 CONTINUE
      BSSY(1)=BSSY1
      BSSY(2)=BSSY2
      DBSSY1=-BSSY(2)
      I=1
   80 I=I+1
      BSSY(I+1)=2.*(I-1)*BSSY(I)/X-BSSY(I-1)
      BSSYI1=BSSY(I+1)
      IF(ABS(BSSYI1).GE.1.E10) GO TO 100
      GO TO 80
  100 NMAX=I+1
      IF(NMAX.GE.N) NMAX=N
      N1=N-1
      CALL BSLJ2(XB,FJ,NMAX+1,0.D00,7,IERR,DUM1,DUM2)
      DFJ1=-FJ(2)
      FM(1)=1./(BSSY(1)**2+FJ(1)**2)
      FN(1)=1./(DBSSY1**2+DFJ1**2)
      DO 200 I=1,N1
      IF(I.GE.NMAX) GO TO 250
      DBSSY=BSSY(I)-I*BSSY(I+1)/X
      DFJ=FJ(I)-I*FJ(I+1)/XB
      FM(I+1)=1./(BSSY(I+1)**2+FJ(I+1)**2)
      FN(I+1)=1./(DBSSY**2+DFJ**2)
  200 CONTINUE
  250 CONTINUE
      N=NMAX
      RETURN
      END
C     SUBROUTINE 'BESY'
C        PURPOSE
C           COMPUTE THE Y BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER
C
C        USAGE
C           CALL BESY(X,N,BY,IER)
C
C        DESCRIPTION OF PARAMETERS
C           X   -THE ARGUMENT OF THE Y BESSEL FUNCTION DESIRED
C           N   -THE ORDER OF THE Y BESSEL FUNCTION DESIRED
C           BY  -THE RESULTANT Y BESSEL FUNCTION
C           IER-RESULTANT ERROR CODE WHERE
C              IER=0  NO ERROR
C              IER=1  N IS NEGATIVE
C              IER=2  X IS NEGATIVE OR ZERO
C              IER=3  BY HAS EXCEEDED MAGNITUDE OF 10**70
C
C        REMARKS
C           VERY SMALL VALUES OF X MAY CAUSE THE RANGE OF THE LIBRARY
C           FUNCTION ALOG TO BE EXCEEDED
C           X MUST BE GREATER THAN ZERO
C           N MUST BE GREATER THAN OR EQUAL TO ZERO
```

```
C
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C         NONE
C
C      METHOD
C         RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE
C         AS DESCRIBED BY A.J.M.HITCHCOCK,'POLYNOMIAL APPROXIMATIONS
C         TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED
C         FUNCTIONS', M.T.A.C., V.11,1957,PP.86-88, AND G.N. WATSON,
C         'A TREATISE ON THE THEORY OF BESSEL FUNCTIONS', CAMBRIDGE
C         UNIVERSITY PRESS, 1958, P. 62
C
C   ..................................................................
C
      SUBROUTINE BESY(X,N,BY,IER)
C
C     CHECK FOR ERRORS IN N AND X
C
      IF(N) 180,10,10
   10 IER=0
      IF(X) 190,190,20
C
C     BRANCH IF X LESS THAN OR EQUAL 4
C
   20 IF(X-4.0) 40,40,30
C
C       COMPUTE Y0 AND Y1 FOR X GREATER THAN 4
C
   30 T1=4.0/X
      T2=T1*T1
      P0=((((-.0000037043*T2+.0000173565)*T2-.0000487613)*T2
     1   +.00017343)*T2-.001753062)*T2+.3989423
      Q0=((((.0000032312*T2-.0000142078)*T2+.0000342468)*T2
     1   -.0000869791)*T2+.0004564324)*T2-.01246694
      P1=((((.0000042414*T2-.0000200920)*T2+.0000580759)*T2
     1   -.000223203)*T2+.002921826)*T2+.3989423
      Q1=((((-.0000036594*T2+.00001622)*T2-.0000398708)*T2
     1   +.0001064741)*T2-.0006390400)*T2+.03740084
      A=2.0/SQRT(X)
      B=A*T1
      C=X-.7853982
      Y0=A*P0*SIN(C)+B*Q0*COS(C)
      Y1=-A*P1*COS(C)+B*Q1*SIN(C)
      GO TO 90
C
C       COMPUTE Y0 AND Y1 FOR X LESS THAN OR EQUAL TO 4
C
   40 XX=X/2.
      X2=XX*XX
      T=ALOG(XX)+.5772157
      SUM=0.
      TERM=T
      Y0=T
      DO 70 L=1,15
```

```
      IF(L-1)50,60,50
   50 SUM=SUM+1./FLOAT(L-1)
   60 FL=L
      TS=T-SUM
      TERM=(TERM*(-X2)/FL**2)*(1.-1./(FL*TS))
   70 Y0=Y0+TERM
      TERM = XX*(T-.5)
      SUM=0.
      Y1=TERM
      DO 80 L=2,16
      SUM=SUM+1./FLOAT(L-1)
      FL=L
      FL1=FL-1.
      TS=T-SUM
      TERM=(TERM*(-X2)/(FL1*FL))*((TS-.5/FL)/(TS+.5/FL1))
   80 Y1=Y1+TERM
      PI2=.6366198
      Y0=PI2*Y0
      Y1=-PI2/X+PI2*Y1
C
C     CHECK IF ONLY Y0 OR Y1 IS DESIRED
C
   90 IF(N-1)100,100,130
C
C     RETURN EITHER Y0 OR Y1 AS REQUIRED
C
  100 IF(N)110,120,110
  110 BY=Y1
      GO TO 170
  120 BY=Y0
      GO TO 170
C
C     PERFORM RECURRENCE OPERATIONS TO FIND YN(X)
C
  130 YA=Y0
      YB=Y1
      K=1
  140 T=FLOAT(2*K)/X
      YC=T*YB-YA
      IF(ABS(YC)-1.0E70)145,145,141
  141 IER=3
      RETURN
  145 K=K+1
      IF(K-N)150,160,150
  150 YA=YB
      YB=YC
      GO TO 140
  160 BY=YC
  170 RETURN
  180 IER=1
      RETURN
  190 IER=2
      RETURN
      END
```

```
C     SUBROUTINE BSLJZ(X , FJ , NMAX , A , ND , IERR , FJAPRX , RR)
C
C THIS IS ONE OF THREE ROUTINES, "BSLJZ", "BSLIZ", AND "BSCJZ",
C BASED ON ALGORITHM 236 FROM "COMMUNICATIONS OF THE A.C.M.",
C AUGUST 1964. THIS ONE EVALUATES THE BESSEL FUNCTIONS OF THE
C FIRST KIND FOR REAL ORDERS AND NON-NEGATIVE REAL ARGUMENTS.
C
C THE PARAMETERS ARE DESCRIBED AS FOLLOWS, WITH "(I)", "(O)", AND
C "(I/O)" INDICATING, RESPECTIVELY, THAT A PARAMETER IS TO BE SET ON
C ENTRY, WILL BE SET BY THE ROUTINE, OR BOTH :
C
C *** ALL PARAMETERS EXCEPT "ND" , "IERR" , "NMAX" ARE ***
C *** DOUBLE PRECISION.                                ***
C
C (I)     X        --  THE (NON-NEGATIVE) ARGUMENT TO THE BESSEL FUNCTIONS.
C (O)     FJ       --  AN ARRAY IN WHICH THE VALUES OF THE BESSEL FUNCTIONS
C                      ARE STORED, AS FOLLOWS: LET J(X;B) DENOTE THE VALUE
C                      OF THE BESSEL FUNCTION OF ORDER B WITH ARGUMENT X.
C                      THEN, FOR I = 1 TO ABS(NMAX)+1,
C                            FJ(I) = J(X;A + (I-1)*SIGN(NMAX)) .
C (I)     NMAX     --  REFER TO "FJ".
C (I)     A        --  REFER TO "FJ". NORMALLY, 0 <= A < 1, BUT THE ALGOR-
C                      ITHM WORKS, WITH SOME LOSS OF ACCURACY, FOR A >= 1.
C                      SEE THE PROGRAM NOTES BELOW.
C (I)     ND       --  THIS GIVES THE NUMBER OF SIGNIFICANT FIGURES OF
C                      ACCURACY DESIRED IN THE FUNCTION VALUES.
C (O)     IERR     --  THIS IS AN ERROR FLAG WHICH IS SET TO 0 IF THE
C                      INPUT PARAMETERS ARE OKAY, AND TO SOME POSITIVE
C                      VALUE IF ONE OF THE PARAMETERS IS INVALID. REFER
C                      TO THE ERROR EXITS AT THE END OF THE CODE FOR A
C                      DETAILED LIST OF THE VALUES OF IERR.
C (O)     FJAPRX   --  A SCRATCH ARRAY USED BY THE ROUTINE. IT MUST HAVE
C                      AT LEAST ABS(NMAX)+1 ENTRIES.
C (O)     RR       --  ANOTHER SCRATCH ARRAY. IT TOO MUST HAVE AT LEAST
C                      ABS(NMAX)+1 ENTRIES
C
C OTHER ROUTINES CALLED: ( * INDICATES A LOCAL ROUTINE )
C  *      NBS01Z -- INVERSE FUNCTION OF X*LOG(X)
C  *      UNDERZ -- ROUTINE TO CONTROL UNDERFLOW INTERRUPTS.
C         DGAMMA -- DOUBLE PRECISION GAMMA FUNCTION.
C         DLOG   -- DOUBLE PRECISION LOGARITHM
C         DABS   -- ABSOLUTE VALUE
C         MOD    -- REMAINDER
C         DMAX1  -- MAXIMUM OF 2 REALS
C
C NOTES:
C         THE METHOD OF COMPUTATION IS A VARIANT OF THE BACKWARD
C         RECURRENCE ALGORITHM OF J.C.P.MILLER (REFERENCE 1). THE
C         PURPORTED ACCURACY IS OBTAINED BY A JUDICIOUS SELECTION
C         OF THE INITIAL VALUE "NU" OF THE RECURSION INDEX (REP-
C         RESENTED IN THE CODE BY THE VARIABLE "XNU"), TOGETHER
C         WITH AT LEAST ONE REPETITION OF THE RECURSION WITH "NU"
C         REPLACED BY "NU"+5. NEAR A ZERO OF ONE OF THE BESSEL
C         FUNCTIONS, THE ACCURACY OF THAT PARTICULAR BESSEL FUNCTION
C         MAY DETERIORATE TO LESS THAN "ND" SIGNIFICANT DIGITS. THE
C         ALGORITHM IS MOST EFFICIENT WHEN X IS SMALL OR MODERATELY
C         LARGE.
C
C         THE ABOVE PARAGRAPH IS TAKEN FROM GAUTSCHI'S PRESENTATION
C         OF ALGORITHM 236 IN C.A.C.M. THE SELECTION OF THE INITIAL
C         "NU" IS DONE WITH THE AID OF THE FUNCTION NBS01Z, ALSO
C         BY GAUTSCHI (AND CALLED "T" BY HIM). IN THIS CODE, THE
C         FOLLOWING SPECIAL CASES HAVE BEEN ADDED:
C                     A. X=0 WHEN NMAX > 0 OR A=0
C                     B. A=0 AND NMAX < 0
C                     C. A >= 1 : THE ALGORITHM WORKS IN THIS CASE, BUT THE
C                                 INITIAL CHOICE OF "NU" IS NO LONGER
C                                 OPTIMAL, AND SOME ACCURACY IS LOST. SIMPLE
C                                 TESTS INDICATE THAT ONLY A FEW DECIMAL
C                                 PLACES ARE SACRIFICED AT WORST. A LIMIT OF
C                                 "ABIG" IS PLACED ON A TO AVOID OVERFLOW IN
C                                 THE GAMMA FUNCTION. TO AVOID COMPLICATIONS,
C                                 NMAX IS REQUIRED TO BE NON-NEGATIVE IF A > 1.
```

```
C REFERENCES:
C       1. GAUTSCHI, W. "RECURSIVE COMPUTATION OF SPECIAL FUNCTIONS",
C              UNIVERSITY OF MICHIGAN ENGINEERING SUMMER CONFER-
C              ENCES, NUMERICAL ANALYSIS, 1963.
C
C*************************************************************************
      SUBROUTINE BSLJZ(X , FJ , NMAX , A , ND , IERR , FJAPRX , RR)
      IMPLICIT REAL*8 (A-H,O-Z)
      DOUBLE PRECISION NBS01Z
      DIMENSION FJ(1) , FJAPRX(1) , RR(1)
      LOGICAL   NEVEN , AFLAG
      DATA      ONE/1D0/          , TWO/2D0/         , HALF/.5D0/      ,
     *          TEN/10D0/         , SMALL/1D-15/     , C1/.73576D0/    ,
     *          C2/1.3591D0/      , C3/2.3026D0/     , C4/1.3863D0/    ,
     *          ZERO/0D0/         , ABIG/55D0/       , TWOP5/2.5D0/    ,
     *          ALEPH/Z7FFFFFFFFFFFFFFF/            , FOUR/4D0/       ,
     *          C5/Z4F10000000000000/
C******
C INITIALIZE THE ERROR PARAMETER , TURN UNDERFLOW OFF , AND CHECK
C THE PARAMETERS FOR VALIDITY AND FOR THESE SPECIAL CASES:
C       A. X≠0 WITH NMAX > 0 OR A=0
C       B. A=0 AND NMAX < 0
C
C THE CODE DELIBERATELY AVOIDS TESTING MORE THEN ONE THING IN EACH
C LOGICAL "IF" BELOW BECAUSE OF I.B.M. FORTRAN INEFFICIENCY IN THIS
C REGARD.
C
C IF A>1, NMAX MUST NOT BE NEGATIVE.
C******
      IERR = 0
      CALL UNDERZ('OFF',SAVE)
      IF(A    .LT. ZERO) GOTO 999
      IF(A    .GT. ABIG) GOTO 998
      IF(X    .LT. ZERO) GOTO 997
      IF(NMAX .GE. 0   ) GOTO 10
      IF(A    .EQ. ZERO) GOTO 10
      IF(A    .LE. SMALL) GOTO 996
      IF(A    .GE. ONE  ) GOTO 994
10    IF(X    .GT. ZERO) GOTO 40
      IF(NMAX .GE. 0   ) GOTO 20
      IF(A    .GT. ZERO) GOTO 995
C******
C IF NMAX < 0, NMAXT IS SET HERE SO THAT ONLY J(X;A) IS CALCULATED.
C THE LOOP FOLLOWING STATEMENT 800 THEN CALCULATES THE REMAINING
C FUNCTIONS BY A SIMPLE RECURRENCE.
C IF A=0, NMAXT IS SET SO THAT J(X;A+N), N=0,...,-NMAX ARE
C CALCULATED; THE CODE AFTER 800 THEN REVERSES THE SIGN OF EVERY
C OTHER ONE.
C
C WE FIRST HANDLE THE CASE X=0.
C******
20    NTEMP = IABS(NMAX) + 1
      DO 30 I = 1,NTEMP
30    FJ(I) = ZERO
```

```
      IF(A .EQ. ZERO) FJ(1) = ONE
      GOTO 1000
C******
40    AFLAG = (A .EQ. ZERO) .AND. (NMAX .LT. 0)
      NMAXT = NMAX
      IF(NMAX .LT. 0) NMAXT = 1
      NTEMP = MAX0(NMAX+1,1)
      IF(.NOT. AFLAG) GOTO 60
      NMAXT = - NMAX
      NTEMP = NMAXT + 1
60    EPSLON = TEN**(-ND)/2
      DO 80 I = 1,NTEMP
80    FJAPRX(I) = ZERO
      SUM = (X/TWO)**A/DGAMMA(ONE+A)
      D1  = C3*ND + C4
      R   = ZERO
      IF(NMAXT .GT. 0) R = NMAXT * NBS01Z(HALF*D1/NMAXT)
      S     = C2 * X * NBS01Z(C1*D1/X)
C******
C THE RECURSION INDEX "NU" IS DELIBERATELY CALCULATED AS A FLOATING
C POINT NUMBER RATHER THAN AN INTEGER, AND ALL COMPARISONS WITH IT
C ARE DONE AS FLOATING POINT COMPARISONS.
C******
      XNU   = ONE + DMAX1(R,S)
      XLIMIT = XNU/2
      TWOA = A + A
      XN    = ZERO
      FL    = ONE
C******
C THE OUTER ITERATION LOOP STARTS HERE.
C
C
C THE FOLLOWING LOOP IS DONE ENTIRELY IN FLOATING POINT FOR
C EFFICIENCY.
C******
200   XN = XN + ONE
      FL = FL * (XN + A)/(XN + ONE)
      IF(XN .LT. XLIMIT) GOTO 200
      OLDFL = FL
      OLDXN = XN
C
      N  = 2*XN
      XN = N
      NEVEN = .TRUE.
      R = ZERO
      S = ZERO
      TEMP1 = TWO/X
C******
C IN THE FOLLOWING LOOP, THE SUCCESSIVE VALUES OF "R" ARE PARTIAL
C FRACTIONS OF A CONTINUED FRACTION.
C******
300   DENOM = TEMP1 * (A + XN) - R
      IF(DABS(DENOM) .LE. SMALL) DENOM = DENOM + SMALL
      R = ONE/DENOM
```

```
      FLMBDA = ZERO
      IF(.NOT. NEVEN) GOTO 400
      FL     = FL * (XN + TWO)/(XN + TWOA)
      FLMBDA = FL * (XN + A)
400   S = R * (FLMBDA + S)
      IF(N .LE. NMAXT) RR(N) = R
      N  = N - 1
      XN = XN - ONE
      NEVEN = .NOT. NEVEN
      IF(N .GE. 1) GOTO 300
C******
      FJ(1) = SUM/(ONE + S)
      IF(NMAXT .EQ. 0) GOTO 600
      DO 500 N = 1,NMAXT
500   FJ(N+1) = RR(N) * FJ(N)
C******
C THE LATEST APPROXIMATIONS ARE CHECKED FOR IMPROVEMENT:
C******
600   DO 800 N = 1,NTEMP
        IF(DABS(FJ(N) - FJAPRX(N)) .LE. DABS(FJ(N))*EPSLON) GOTO 800
        DO 700 M = 1,NTEMP
700     FJAPRX(M) = FJ(M)
        XN      = OLDXN
        FL      = OLDFL
        XLIMIT = XLIMIT + TWOP5
        GOTO 200
800   CONTINUE
      IF(NMAX .GE. 0) GOTO 1000
C******
C IF NMAX<0, WE HAVE FINISHED OBTAINING J(X;A)  , AND NOW
C ITERATE TO FIND ALL THE DESIRED FUNCTIONS.
C
C FIRST WE CHECK FOR THE SPECIAL CASE A=0.
C******
      IF(.NOT. AFLAG) GOTO 850
      NMAXT = -NMAX + 1
      DO 820 N = 2,NMAXT,2
820   FJ(N) = - FJ(N)
      GOTO 1000
C******
850   FJ(2) = TWO * A * FJ(1)/X - FJ(2)
      IF(NMAX .EQ. -1) GOTO 1000
C******
C THE FOLLOWING CODE IS A RENDITION OF THE LOOP
C                  DO 900 N = 2,NMAXT
C        900       FJ(N+1) = (2/X)*(A-N)*FJ(N) - FJ(N-1)
C
C WITH OVERFLOW DETECTION. AS SOON AS THE NUMBERS GET TOO BIG, THEY
C ARE SCALED DOWN (BY A POWER OF THE MACHINE BASE, SO AS TO AVOID
C LOSS OF PRECISION) AND THE CALCULATION CONTINUES. A SEPARATE LOOP
C TRANSFORMS THE SCALED VALUES TO THE CORRECT OUTPUT VALUES, SETTING
C TOO-LARGE ONES TO PLUS OR MINUS INFINITY.
C******
      NMAXT = -NMAX + 1
```

```
      FJNM2 = FJ(1)
      FJNM1 = FJ(2)
      TEMP1 = TWO/X
      OVER  = ZERO
      XNM1  = TWO
C
      DO 880 N = 3,NMAXT
      FJN   = TEMP1 * (A - XNM1) * FJNM1   -   FJNM2
      FJNM2 = FJNM1
      FJNM1 = FJN
      FJ(N) = FJN
      XNM1  = XNM1 + ONE
      RR(N) = OVER
      IF(DABS(FJN) .LT. C5) GOTO 880
      OVER  = OVER + ONE
      FJNM1 = FJNM1/C5
      FJNM2 = FJNM2/C5
880   CONTINUE
C
      IF(NMAXT .LE. 3) GOTO 1000
      OVER  = ZERO
      SCALE = ONE
C
      DO 900 N = 4,NMAXT
      IF(OVER .LT. FOUR) GOTO 890
      FJ(N) = DSIGN(ALEPH,FJ(N))
      GOTO 900
890   IF(RR(N) .GT. OVER) SCALE = SCALE * C5
      FJ(N) = FJ(N) * SCALE
      OVER  = RR(N)
900   CONTINUE
      GOTO 1000
C*******
C ERROR EXITS FOLLOW. MEANINGS OF THE EXIT VALUES OF "IERR" ARE:
C          0 : NO ERROR
C          1 : A < 0
C          2 : A > ABIG
C          3 : X < 0
C          4 : NMAX < 0 AND 0 < A < SMALL
C          5 : X=0, NMAX < 0, AND A > 0
C          6 : NMAX < 0 AND A >= 1
C*******
994   IERR = IERR + 1
995   IERR = IERR + 1
996   IERR = IERR + 1
997   IERR = IERR + 1
998   IERR = IERR + 1
999   IERR = IERR + 1
1000  CALL UNDERZ('S',SAVE)
      RETURN
      END
```

End 9-77